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Response analysis of beef feedlot gain with respect to genetic and environmental inputs

Bryan Edwin Melton
Iowa State University

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Response analysis of beef feedlot gain with
respect to genetic and environmental inputs

by

Bryan Edwin Melton

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
MASTER OF SCIENCE

Departments: Economics
Animal Science

Co-majors: Agricultural Economics
Animal Breeding

Approved: _____

Signatures have been redacted for privacy

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Ames, Iowa

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CHAPTER I. INTRODUCTION

Development of the Problem

In the recent past, the beef industry has undergone changes amounting to a revolution. Most recently, these changes have involved an increased emphasis on the economic aspects of production. All areas of animal science have been subjected to a deluge of questions and demands for knowledge concerning costs and profits.

In animal breeding, these questions cover the spectrum of production. Producers, be they cow-calf operators or feedlot managers, have become increasingly concerned with the economic considerations of breeding. Producers want to know answers to such questions as: What is the most profitable breed? What is the value of selection in dollars and cents? How much selection can I afford? What is the value of progeny testing? What is the best animal? While these questions may be relevant to all phases of the industry, any answer must be directed to a specific phase. For example, the most profitable animal for a feedlot operator may not be the best for the cow-calf producer.

As animal breeders attempt to answer these questions, an increased utilization of economic principles in the analysis is required. The incorporation of two disciplines into one framework is never easy. When these two represent a physical and a social science, it is even more difficult. Because of the problems of reconciling these areas and the very

recent increased emphasis toward economics, a great many questions remain either totally or partially unanswered.

A model is needed which, with the application of sound economic and animal science principles, will yield greater insight into the problems facing the beef industry. Such a model should allow producers to evaluate animals in economic terms based on their genetic composition. This could lead to a reevaluation of selection efficiency, breeds, and traits based on a dollar value to the industry. This paper presents just such a model and some of the relevant implications for the feedlot phase of the industry.

Objective of the Study

While this paper is specifically concerned with only the feedlot or postweaning phase of the industry, the methodology can easily be extended to all segments of production. In general, therefore, the objective may be stated as the incorporation of principles of animal science and economics into a single model which will approximate situations confronting producers. Specifically, this model will be concerned with only the feedlot segment and will attempt to consider such areas as the value of a trait, the optimal breed, the optimal animal, and the demand for genetic ability in selected traits. Beyond these specifics, policy implications for breed associations, cow-calf producers, and market structure can be considered as well.

CHAPTER II. PROCEDURES

Relevant Theories

Before an empirical analysis of any sort can be undertaken, an understanding of the underlying theory is essential. When there are two disciplines involved, the problem becomes threefold. First, an understanding of at least the rudiments of each field's theory is required, then the two must be combined.

In the interest of providing some understanding of the relevant theories, a very abbreviated and oversimplified presentation is made. Oversimplified is stressed, but it is hoped it will provide some understanding to the uninitiated.

The theory of the firm

The firm can be basically defined as an economic agent which is involved in production. Production is further defined as the process of transforming inputs into outputs. This transformation is accomplished subject to technical rules specified by the relevant production function. Since the production function lies at the heart of the theory of the firm, it will be necessary to consider it in more detail.

Production functions, as already described, govern the transformation of inputs into outputs. In more general terms, this entire area of knowledge might be referred to as response analysis. As the name implies, response analysis concerns the analysis of output magnitude in response

to a set of inputs. This analysis may take either a positive or a normative approach, where the positive approach is concerned with the nature of production itself and the normative addresses itself to problem solving.

Positive response analysis The initial step of a positive analysis is the realization that many levels of output are available from the same inputs. In other words, inputs at a fixed level can produce many levels of output. Of these many levels of output there is one, unambiguous maximum. This maximum, and all output levels below this maximum form the production set. The production function forms the upper boundary of this set. From this presentation it is apparent that the production function presupposes a maximum. Other points or nonmaximums are generally of little interest and are neglected. It is assumed that they have arisen through inefficiency or waste.

By considering an implicit function such as equation (1), general properties of production functions can be considered. In this representation, Y is the output and X_i is the i th input. Typical economic theory generally dictates that this function have three general properties which may be summarized as follows:

$$Y = f(X_1, X_2, \dots, X_i, \dots, X_n) \quad (1)$$

1. The resulting surface is smooth and continuous between inputs and outputs;
2. diminishing marginal products prevail with respect to all inputs; and
3. decreasing returns to scale exist (4, 18).

The first condition insures the existence of the first partial derivative of the function. The second and third conditions insure the direction of the second partial derivation and the fact that less than proportional increases in output result from proportional increases in all inputs.

Given a function which obeys the specified properties, it is possible to compute average productivities, marginal productivities, elasticities of production, elasticities of substitution, and marginal rates of substitution between inputs.¹ The marginal rate of substitution is an important concept. First, it should be at least intuitively obvious that a given output level can be produced by various levels of the same inputs. For example, cultivation and pesticide use can be varied against each other to produce the same yield. The different combinations of inputs producing the same level of output are referred to as isoquants, and the slope of these isoquants is simply the marginal rate of substitution. All points on different isoquants with the same marginal rate of substitution form isoclines. Two special isoclines are those where the marginal rate of substitution equal zero or infinity and are called ridgelines. These lines form the boundary of the isoquant set. Graphically, these concepts are presented with two inputs in Figure 1. The properties of production functions outlined further insure that this isoquant set is continuous and convex (18, 32).

¹The reader is referred to the list of terms and abbreviations for definitions of unfamiliar terms.

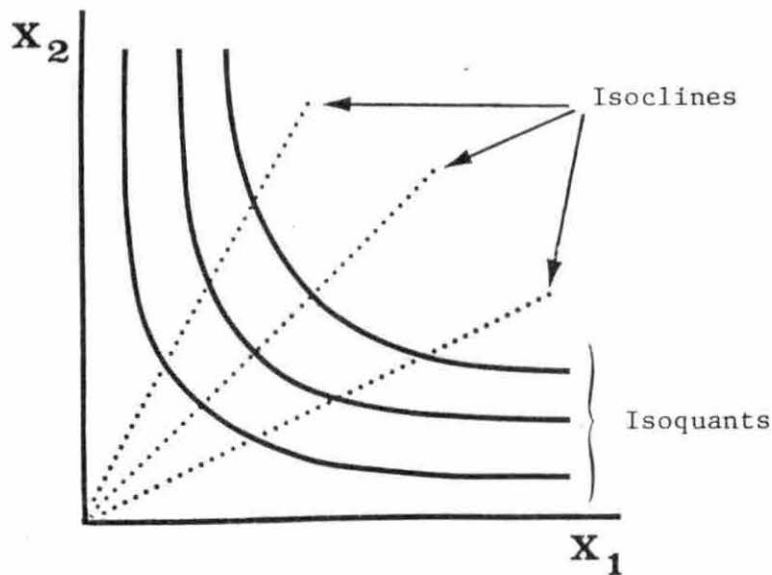


Figure 1. Isoquants and isoclines for a typical production function with two inputs (X_1 and X_2)

Normative response analysis With these properties developed, the next logical step is the normative or problem-solving analysis. This analysis requires some further development of the theory of the firm to define the problem to be solved.

First, some objective must be defined for each firm. In some cases, the firm might maximize sales, in others it might maximize returns or profits. While the latter is the most frequent case in economic theory, there is some indication that others do exist. Since maximizing profit is the most frequently used and the most accepted criteria, other criteria will be considered only in passing.

Profit can be defined as total revenue less total costs. When these costs are composed of variable and fixed costs, the profit may be expressed as

$$\pi = TR - TC = TR - (VC + FC). \quad (2)$$

As total revenue is a function of the quantity and price of output, and variable costs are the sum of inputs used multiplied by their respective cost per unit, equation (2) may be rewritten as

$$\pi = P_y Y - \sum_{i=1}^n P_{x_i} X_i - FC. \quad (3)$$

where:

π = profit;

P_y = price received per unit of output;

Y = number of units of output;

P_{x_i} = price paid per unit of the i th input; and

X_i = number of units of the i th input.

Furthermore, since the output, Y , is determined by the relevant production function, substitution yields:

$$\pi = P_y \left[f(X_1, \dots, X_i, \dots, X_n) \right] - \sum_{i=1}^n P_{x_i} X_i - FC. \quad (4)$$

Maximizing this function with respect to X_i yields

$$\frac{\partial \pi}{\partial X_i} = P_y f'(X_i) - P_{x_i} \quad i = 1, \dots, n \quad (5)$$

By setting equation (5) equal to zero, the conditions for profit maximization are obtained:¹

$$P_y f'(X_i) = P_{x_i} \quad i = 1, \dots, n \quad (6)$$

Since $f'(X_i)$ is the first partial derivative of the production function with respect to the i th input, it represents the marginal productivity of the i th input. The marginal conditions of equation (6) requires that each input be used up to the point where the value of the marginal product (MVP) equals the input cost. The value of the marginal product is the price of output times the marginal product and in the case of competition, the input cost is the factor marginal cost (FMC). The marginal conditions then equate the value of the marginal product with the factor marginal cost. While this is an admittedly simplified explanation, it does serve to illustrate the properties of firms, the function of prices, and a normative analysis of response.

By again considering only two inputs, the results of this normative analysis can be related to Figure 1 of the positive analysis. Specifically, dividing the maximizing condition of one input (X_1) by the other (X_2), yields equation (7). The right-hand side of this equation is the inverse of the marginal rate of substitution of X_1 for X_2 . Therefore, by inverting the entire equation, the condition that the inverse price ratio equals the marginal rate of substitution is realized.

¹Second-order conditions are assumed to hold. Explanation of these conditions may be found in any general microeconomics text including books by Henderson and Quandt (18) and Stigler (32).

$$\frac{P_{X_1}}{P_{X_2}} = \frac{P_y f'(X_1)}{P_y f'(X_2)} = \frac{f'(X_1)}{f'(X_2)} \quad (7)$$

Graphically, the negative inverse price ratio represents the slope of some budget line and optimal occurs at the tangency of this line and the isoquant. This situation is represented in Figure 2 by imposing a budget line on Figure 1.

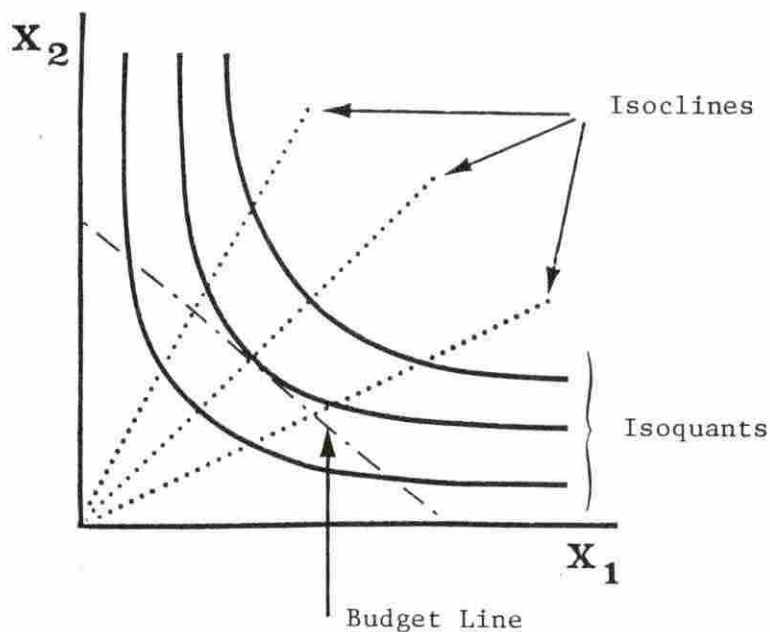


Figure 2. Isoquants and isoclines for a typical production function with two inputs (X_1 and X_2) illustrating the imposition of a budget line

As an alternative to profit maximization, consider a firm which maximizes production. In this case, the normative analysis assumes a function such as equation (8).

$$\gamma = Y = f(X_1, \dots, X_i, \dots, X_n) \quad (8)$$

It should be obvious that equations (8) and (4) yield the same answer only when $P_y = 1$ and $P_{x_i} = 0$ for all X_i . In other words, inputs are free. This is the primary reason that such objectives as maximizing production are economically unrealistic.

Population genetics

Since there are many aspects to the manner in which genes function in a population, or population genetics, it is necessary to consider a very limited viewpoint. In this respect, the manner in which gene frequencies change will be primarily restricted to selection. Selection is one way in which a producer can change a herd in a manner predictable in both direction and amount. In addition, some attention will be given to the effects of inbreeding and crossbreeding. Since in population genetics, traits which exhibit continuous variation are analyzed, three basic relationships will be used in this analysis: means, variances, and covariances.

Mean values in selection Initially, consider observing a single trait in a herd of cattle, and that this trait exhibits continuous variation, such as weight. The measurements taken on this trait are referred to as the phenotypic value, P . This phenotypic value is actually the result of some inheritance from the parents and the effects of the environment. The inherited part of P is referred to as the genotypic value, G , and the environment as environmental deviations, E . In other words, the phenotypic value is the result of the genotypic value and deviations caused by environment. Symbolically this relationship can be represented as

$$P = G + E. \quad (9)$$

The genotypic value is the result of genes received from the parents, with each parent contributing one-half of the genes in the progeny. As the genes are recombined in the progeny, they produce a genotype which is then assigned some value to result in genotypic value. If the genes are symbolized as A_1 and A_2 , then this relationship may be illustrated in Figure 3 as done by Falconer (6, p. 113).

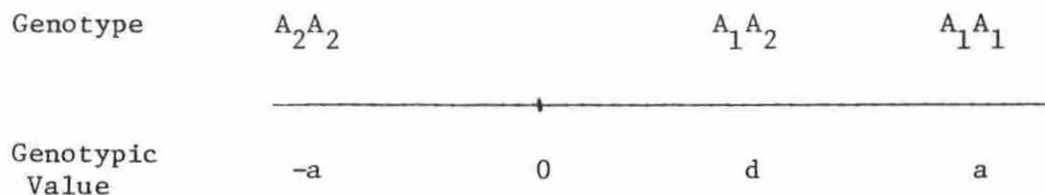


Figure 3. Assignment of relative genotypic values by genotype

In this presentation, A_1 is a dominant gene and A_2 is recessive. The values are arbitrary, but the position of the heterozygote,¹ A_1A_2 , depends on the degree of dominance for A_1 over A_2 . For example, if A_1 is completely dominant, $d = a$.

It can now be seen that the mean genotypic value of a population is a function of the value of a genotype and the frequency of that genotype's occurrence in a population. If p = the frequency of A_1 and q = the frequency of A_2 , and the sum of the frequencies equals one, then the

¹The reader is referred to the list of terminology and abbreviations for unfamiliar terms (Appendix B).

population mean genotypic value (M) for a randomly mating population, may be expressed as

$$M = a(p-q) + 2dpq \quad (10)$$

This result is obtained by observing that if the frequency of $A_1 = p$ then the frequency of $A_1A_1 = p^2$, and remembering that there are two ways to obtain the heterozygote, A_1A_2 and A_2A_1 . The frequency of each genotype multiplied by its value and summed over all genotypes yields the population mean genotypic value.

The next item one might consider is the deviation of a certain animal's progeny from this mean. This value, termed breeding value, is the sum of the average effect of the genes possessed by the individual. These effects are referred to as the average effect of a gene substitution. By determining the mean genotypic values of the substitution and subtracting M , the average effect of gene substitution can be seen. Averaging the effects over all genotypes yields

$$\alpha = a + d(q-p) \quad (11)$$

where α = the average effect of gene substitution.

In order to reach a breeding value, each genotype must be considered individually. The breeding values for the genotypes previously defined are presented as follows:

Genotype	Breeding Value
A_1A_1	$2q\alpha$
A_1A_2	$(q-p)\alpha$
A_2A_2	$-2p\alpha$

The average breeding value, A, is again the sum of the breeding values, for all loci. This value is sometimes called the additive genotypic value.

Thus, a portion of G has been explained as the breeding value of the progeny. The remainder of G is referred to as dominance deviations, D. Nothing will be done with D except to state that it represents genetic interactions. The resulting information may then be expressed as

$$G = A + D \quad (12)$$

Variability of values The next property one would consider in observing this herd is the variance of measurements. This may be symbolized as

$$V(P) = V(G) + V(E) = V(A) + V(D) + V(E) \quad (13)$$

where:

V(P) = variance of P;

V(G) = variance of G; etc.

This presentation assumes there is no correlation between genotypic values and environmental deviations and is usually a satisfactory representation.

These variances lead to one of the most important concepts in population genetics, namely heritability. Heritability may be defined as the portion of phenotypic variance, V(P), due to additive genetic variance, V(A). The ratio may be estimated by direct observation of V(P) and observation of the similarity between relatives to estimate V(A), because they have genes and consequently, effects in common.

$$h^2 = \frac{V(A)}{V(P)} = \beta_{AP} \quad (14)$$

where β_{AP} = the regression coefficient of A on P.

Heritability finds one of its greatest uses in selection. As previously stated, this selection progress is predictable in both direction and amount and concerns the selection of individuals to be used as parents. The greatest interest in selection is in the changes which occur in the population mean. This change is termed response and symbolized by R. This response may be achieved by selecting superior animals as parents, where S, or the average superiority of selected parents, is called the selection differential. It should be remembered that this selection must be accomplished on phenotypic values as they are all that can be observed. However, the relationships previously developed form the connection between the phenotypic and genotypic values.

It should be apparent that response is due to both the superiority of the parents and their ability to pass their superiority on to the next generation, or

$$R = h^2 S. \quad (15)$$

If the relationship is standardized by dividing through by σ_p , or the standard deviation of phenotypic values, the equation may be written as

$$R = ih\sigma_A. \quad (16)$$

This revision is made by noting the definition of heritability given in equation (14) and defining

$$i = S/\sigma_p \quad (17)$$

as the intensity of selection.

Correlation of values Thus far, only one trait has been considered, whereas in reality the producer is concerned with several. In the case of more than one trait, the values may be correlated and should be considered in making selections. These correlated responses may be referred to as genetic correlation or the correlation of breeding values by estimating the likeness among relatives.

This correlation is explained by considering two traits, X and Y. The response of X to selection is the same as if X were the only gene as in equation (16). To determine the correlated response in Y, it is necessary to know the correlation of breeding values as well. The correlated response of Y then becomes

$$CR_Y = ih_x \sigma_{A_Y} r_A \quad (18)$$

where:

CR_Y = correlated response in Y;

h_x = square-root of heritability of X;

σ_{A_Y} = standard deviation of additive genetic value of Y; and

r_A = genetic correlation of X and Y.

In the case of selecting for more than one trait, the correlated responses and heritabilities have led to the use of relative scores or indexes (3, 6, 9, 24). These indexes must be determined by weighting each trait in some appropriate fashion. At this point a very abbreviated presentation of this relationship is required.

First, the aggregate breeding value of n traits, H , is defined as the sum of the weighted individual breeding values:

$$H = \sum_{i=1}^n a_i A_i \quad (19)$$

where a_i is the weight of the i th trait and A_i is the breeding value of the i th trait. The index itself, I , may be defined as the sum of some other weight b_i times the phenotypic value of a trait P_i :

$$I = \sum_{i=1}^n b_i P_i \quad (20)$$

By choosing b_i such that it maximizes the correlation between H and I , r_{HI} , and converting to matrix notation, the result is that

$$b = P^{-1} Aa \quad (21)$$

where:

b = a $n \times 1$ vector of weights;

P = a $n \times n$ matrix of phenotypic covariances;

A = a $n \times n$ matrix of genotypic covariances; and

a = a $n \times 1$ vector of weights.

Specifically, if selection is to be concerned with economic progress, a is a vector of economic weights relating the added profit due to the considered traits. The transformation from breeding and phenotypic values is made by relationships of heritability, equation (14), and the properties of correlated responses (18).¹ Furthermore, it should be at least intuitively clear that the weights, b , are analogous to a weighted heritability, i , in a one trait case.

¹More detail on the necessary derivation is available in Falconer (6) as well as other sources, but is not considered essential to the understanding of the concepts.

Effects of breeding system: crossbreeding and inbreeding

Crossbreeding and inbreeding are concerned with the manner in which the selected parents are combined. Inbreeding refers to crossing individuals more closely related to each other than an average pair in the population from which they were selected. Crossbreeding is, of course, the opposite case. In general, inbreeding causes lines within a population to form.

The variance of genotypes within these lines is reduced due to the chance loss or fixation, of certain genotypes. One result of inbreeding is an increase in the homozygotes, A_1A_1 and A_2A_2 , at the expense of the heterozygotes. In general, inbreeding results in a decrease in the fitness of an individual because the A_2A_2 is considered less desirable than A_2A_1 .

Crossbreeding has opposite effects and restores the loss of fitness in one generation. The restoration is termed heterosis and is defined as the difference between the average of the reciprocal crosses and the average of the parent groups. The heterosis is then $(\Delta p)^2 d$. This means it depends on the degree of dominance, d , and the square of the change in frequency.¹

While these concepts, both genetic and economic, are presented in a very naive fashion, it is hoped they will form a base for mutual understanding. From this base, the two fields can be combined.

Combination of the theories

The first step in combining these two theories is the definition of the relevant firm. In this paper, the firm will be restricted to a

¹For a further discussion of crossbreeding, see Willham (40).

postweaning or feedlot operation as previously specified. More specifically, this firm may be considered as handling either concentrate or roughage fattening of beef steers with all animals sold for slaughter. In keeping with the theory of the firm, this firm maximizes profit.

The next step is the determination of the relevant production function. This positive analysis requires a careful definition of the inputs and outputs at its conception. Considering output first, the feeder system must be analyzed. The typical feeder buys steers at a certain weight, feeds them a certain feed for a period of time, and eventually sells them at an increased weight. It can then be reasoned that the product being marketed is the weight change put on by the feeder. This will be referred to simply as gain.

The next logical step is the definition of inputs. The feeder uses the typical inputs of labor and capital in the production, but others are of more interest, especially in reference to animal breeding.

From an animal breeding viewpoint, the animal used in the production of gain is of interest. This animal might be regarded as a capital investment itself, much as a machine in other industries. The exception is that this machine is consumed in the production process. The animal represents a variety of traits which are combined with environmental inputs in the production. This idea is very similar to equation (9) and the two concepts are thus combined. Specifically, the gain can be expressed as a function of the genotypic values of the animal and the environment such as

$$g = f(G,E) \quad (22)$$

where g = gain.

While the function might be left in this form, greater information can be provided if G and E are further broken down. Consider specifically the environmental component first. From the statement of the feed process, two items come to immediate light. These two inputs are the feed provided and the time the animals are on feed.

In segregating G , each producer must make his own evaluation, but if traits which are not highly correlated are chosen more information is available from the analysis. The traits chosen should be those suspected of being of the greatest economic value. In other words, traits such as color of hair should be discontinued -- first, because little information on their value would be provided, and their functional form would be difficult to empirically estimate; and second, increased selection pressure on such traits reduces the pressure which can be placed on the more important traits (24).

Beyond the correlation and value problems, heritability is also significant. In other words, the feedlot operator considers the same parameters in evaluating feedlot steers that are considered in forming a selection index for the breeder.

The two disciplines are then combined by defining genetic inputs in relation to the theory of the firm. This result may be shown mathematically as

$$\pi = P_g f(G_1, G_2, \dots, G_i, \dots, G_n, E_{n+1}, E_{n+2}, \dots, E_{n+m}) - \sum_{i=1}^n P_{G_i} G_i - \sum_{i=n+1}^{n+m} P_{E_i} E_i \quad (23)$$

where:

G_i = ith genetic input; and

E_i = ith environmental input.

The profit maximizing conditions are the same as equation (6). One slight addition to the analysis is the breed effect. This is comparable to the crossbreeding (inbreeding) aspects of animal breeding. The most reasonable manner of handling this concept in the theory of the firm is the assumption that one of the genetic inputs, G_i , represents a classification variable for breeding. This procedure allows comparison between breeds as well as within breeds.

The Data

All of the preceding combinations of theories rest basically on the nature of the production function. The data must therefore contain information of a relevant genetic nature, and must provide the basis for empirical estimation of the production function.

The data used for this purpose is the result of a series of experiments conducted at Fort Robinson, Nebraska to study heterosis (20). The experiment was initiated in 1960 using cows of the Hereford, Angus and Shorthorn breeds. The cows were randomly divided into twelve breeding groups, or four groups per breed. The resulting groups were then crossed with sires representing the three breeds such that, basically, twelve sires were used

per year. This scheme resulted in twice as many females being bred to a sire of the same breed as were bred to any one of the other breeds. The breeding then resulted in nine lines of crosses being produced as shown in Table 1.

Table 1. Relative frequency of breed lines by cross

<u>Breed of Dam</u>	<u>Breed of Sires</u>		
	Hereford	Angus	Shorthorn
Hereford	2/12	1/12	1/12
Angus	1/12	2/12	1/12
Shorthorn	1/12	1/12	2/12

Each year a new group of sires was selected as representative of their breed. This allowed progress over the time of the experiment to parallel the industry and minimized inbreeding.

Full records were maintained on progeny produced by the crosses including birth weight within 24 hours of birth. All male progeny were castrated and eventually sold as slaughter steers. Heifers were retained for replacement as needed.

Of the steers produced, a group in the same relative proportions as the breed lines were selected to be individually fed and eventually slaughtered. These steer records, including individual feed consumption and carcass data, were available for 1961 through 1965 calf crops. The steers were treated as identically as possible both within and between years. The one exception was that the years 1964 and 1965 were comprised of 10 28-day feeding periods while those previous had 9. All steers

received a ration of approximately 70 percent TDN on an individual basis.

A typical ration schedule for an entire feeding period is as follows:

- 1) During the first six and one-half periods on feed,

Corn	45 percent
Beet pulp	12 percent
Alfalfa pellets	30 percent
Molasses	7 percent
Soybean	5 percent
Trace mineral salt	1 percent

and during the first period an amount of loose alfalfa hay amounting to 81 pounds per head.

- 2) During the remainder of the feeding period

Corn	35 percent
Oats	20 percent
Beet pulp	20 percent
Alfalfa pellets	10 percent
Molasses	7 percent
Soybean	7 percent
Trace mineral salt	1 percent
Bone meal	5 percent

and supplemental hay of 2.7 pounds per day per animal.

For purposes of easing the eventual statistical estimation, and providing the maximum information on a given steer, further selection within the steers is advised. First, steers which did not complete the feeding trial, i.e., death losses, are not considered. Second, steers which represent the sole within-line record of a sire are not considered. This excludes steers with minimal family information such as those where records are not available on either male or female within-line half-sibs. This is especially true of female half-sibs as their records are many years long, and are, therefore, very important. This selection had the

effect of removing very few of the animals in the concerned lines and resulted in the distribution reflected in Table 2.

Methodology

Linear programming

After the production has been estimated, the problem of analyzing this function from a normative viewpoint arises. Given that most production functions are complex mathematically, this is no easy problem to solve. In the case of a function involving genetic inputs, it becomes more complex in that there is no market for genetic inputs and, therefore, no market price.

This may be seen more clearly by considering once again the marginal conditions for profit maximization expressed in equation (6). The condition requires that the value of the marginal product equal the factor marginal cost. If the i th input or factor is a genetic variable, then on first inspection the fact that its cost or price is unknown appears to be little problem. However, two aspects of the problem make it much more complicated. First, the equation involving the genetic input is only one of a set of equations. If there are n inputs, then there are n such equations in the set and must all be solved simultaneously. Second, the nature of most production functions is such that the value of the marginal product or left-hand side of equation (6) is itself rather complex. For example, if P_y , the price of output, is given, the marginal product of the i th input, MP_i , must still be determined. Examination reveals that

Table 2. Distribution of selected male progeny by breeding line

Line	Line Code	Number of Steers	Number of Sires	Mean	Progeny per Sire	
					High	Low
Hereford	H	47	12	3.92	7	2
Hereford x Angus	HA	17	10	1.70	3	1
Hereford x Shorthorn	HS	24	9	2.67	4	1
Angus x Hereford	AH	28	10	2.80	4	1
Angus	A	46	12	3.83	6	1
Angus x Shorthorn	AS	28	11	2.55	4	1
Shorthorn x Hereford	SH	39	11	3.55	4	1
Shorthorn x Angus	SA	32	11	2.91	2	1
Shorthorn	S	51	11	4.64	9	3
Overall		312		3.22	9	1

MP_i is usually a function of not only the level of the i th input, but the level of usage of the other $n-1$ inputs as well.

One method of handling this complexity is suggested by the first problem mentioned, simultaneous equations. Special techniques have been developed to handle just such situations; the most frequently used of which is linear programming. The key word then becomes "linear."

There is no reason to suspect that the relevant production function is linear, and strong reason to believe it is not. In order to reconcile this difference, it is necessary to approximate the nonlinear function as a series of linear segments. The more linear segments used, the closer the approximation will be to the actual function. This would imply the use of a very large number of linear segments, however, this is usually both unnecessary and in practice nearly impossible.

An estimate of the number of linear segments to be used can be made visually. This estimate is made by first plotting the relevant production function. By superimposing linear segments over this plot, and counting the segments, an estimate can be made. This procedure is then repeated for isoquants to further linearize the production surface. An illustration of this rough estimation procedure can be seen in Figures 4 and 5.

These figures indicate that the number of production levels required to evaluate the function is equal to the number of line segments necessary to linearize it. They also indicate that in the case of more than one input the number of combinations of input levels necessary to

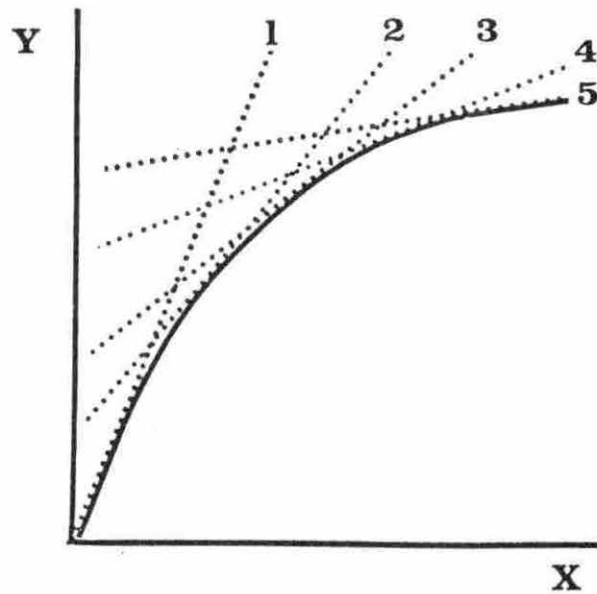


Figure 4. Sample estimation of a nonlinear production function in five linear segments

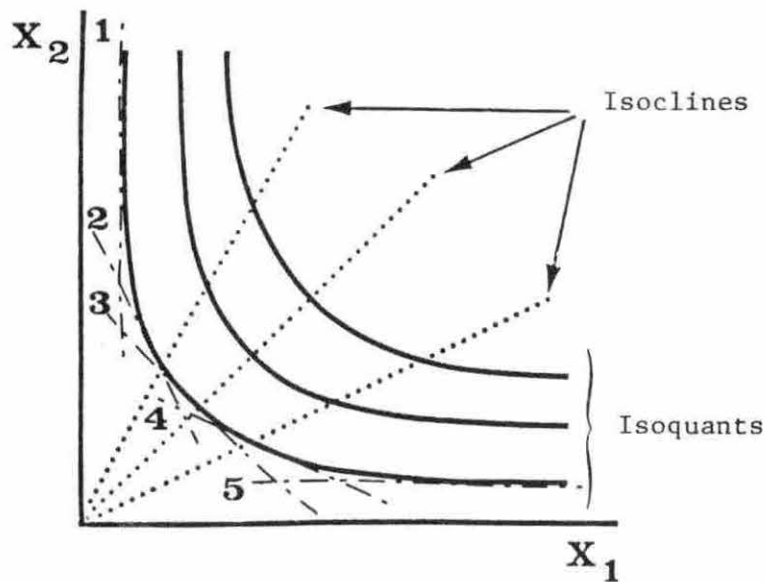


Figure 5. Sample estimation of nonlinear isoquants in five linear segments

evaluate a given production level is equal to the line segments necessary to approximate an isoquant.

While this solves the problem of putting a nonlinear problem into a linear programming framework, there still exists the problem of prices for genetic inputs. Actually, this problem is also solved by the solution technique used in linear programming. Consider for a moment a firm which attempts to maximize profit from a production process using just two inputs. The entrepreneur wishes to know how much production will maximize his profit, given he has a fixed amount of the two inputs to use.

Mathematically his situation may be expressed as

$$\text{Max } \pi = CX \quad (24)$$

subject to

$$a_1 X \leq b_1 \quad (25)$$

$$a_2 X \leq b_2 \quad (26)$$

$$X \geq 0$$

where:

π = profit;

C = net return per unit of output;

X = number of units of output;

a_1 = number of units of input 1 required per unit of output;

a_2 = number of units of input 2 required per unit of output;

b_1 = number of units of input 1 available; and

b_2 = number of units of input 2 available.

Since the expressions, inequalities (25) and (26), cannot be directly solved, a disposal or slack activity is added to the left-hand side of each. This disposal both accounts for unused resources and transforms the inequalities into equalities.

$$a_1X + s_1 = b_1 \quad (27)$$

$$a_2X + s_2 = b_2 \quad (28)$$

In the final solution, either all of a resource is used or the excess is in the disposal activities. Since these disposal activities have a price, C , of zero, they add nothing to the value of the program.

However, if all of one or both resources is used, then there is a shadow price on these slack activities. This shadow price may be defined as the amount by which the value of the program would change if one unit of the exhausted resource were taken out of production and placed in disposal or slack. Intuitively, this value results from decreasing production and the subsequent loss in profit. The shadow price then represents the value of the foregone production due to this unit of scarce resource. This concept is, therefore, analogous to the concept of value of marginal product, MVP.

From this very simplified example, it can be seen that if genetic inputs are treated as resources, a shadow price can be attained. Since this shadow price is equivalent to the value of the marginal product, which is equated to factor marginal cost, FMC, the cost of genetic inputs is determined. This determination, it must be remembered, assumes pure competition, which is a close approximation of agriculture.

Alternative procedures

No one step of the procedure thus outlined may be considered unique. The uniqueness arises in the procedure taken collectively. Previous studies in animal breeding economics have combined the two fields in such a manner that one or more of the aspects presented in this study is missing. This is not to imply that this absence negates the results of these previous studies, only that they are different in some way. As a guide to the alternative procedures available, a limited review of these studies is necessary.

One alternative to the combination presented is the use of the net present value concept. This approach is presented by the Meat and Livestock Commission (25). The analysis is concerned with the net present value, or discounted return, from a superior as opposed to an average herd. This concept is, therefore, comparative and does not imply anything about optimal or profit maximizing.

A second alternative is the net merit approach used by Swiger, et al. (35) and more recently, by Dickerson, et al. (3). This method involves first the definition of net merit as an index of profitability under a specified set of conditions. For example, the study presented by Dickerson assumes four feed situations including feedlot only conditions and integrated cow-calf and feedlot operations. The situations are then related to net merit. Two differences arise in this type of analysis. First, it is again a comparative approach and secondly, the range over which the results apply is based on the differences in the

situations evaluated. For example, all situations assume the feeder faces constant margins, or that feed and beef prices vary proportionally. If this is not the case, the results may not be expected to apply.

Another alternative is the simulation model presented by Fitzhugh, et al. (8) and by Long, et al. (23). The concepts of the theory of the firm and production functions are incorporated with one exception. Again, this is a comparative model and profit maximization or optimization is missing. As in all comparative type models, inferences about optimal can be made only after a very large number of comparisons, approaching infinity.

While the previous three studies are all comparative in nature, the study done by Gibson (9) is optimizing. This study is founded on a linear programming optimization of a swine operation. Different systems are considered as in the net merit approach, but from a range rather than comparative viewpoint. This model differs primarily in the lack of an empirically estimated production function. This implies activities representing an isolated point on the production surface rather than a range of points.

The lack of positive response analysis or empirically estimated functions is not unusual; very few exist using genetics. One example is a study done by Heady, et al. (14). This study estimated the production function for milk production in dairy cattle using both environmental and genetic inputs. A normative, profit maximizing analysis of the function is also presented, but the analysis is accomplished without the use of a linear programming technique.

All this should not be taken to imply that these studies are inferior, only different. Each study may have been conducted with totally different objectives in mind.

CHAPTER III. RESULTS

The Production Function

The analysis of a production function can be broken into three basic parts. The first of these is the careful definition of the variables to be considered. The second part constitutes the selection of an appropriate form and its empirical estimation, and the third is the analysis of the resulting function.

The variables

The initial step in empirically estimating a production function, even before consideration of biological and functional compatibility, should be careful attention to the definition of variables. Thus far, several variables have been alluded to, but formalization of the definitions has been neglected. Other variables, specifically genetic variables, have been left untouched. Since one would expect the genetic variables to present the greatest difficulty, the nongenetic or environment can be specified first.

The environmental variables have already been mentioned, and only a formalization of the definition is needed. The first of these variables is gain, and will be defined as the weight change in kilograms over the entire feeding period. Since the feeding period can be of various lengths, this does not restrict the definition. The gain exhibited by the animal is then the output or dependent variable.

The next consideration of the typical operator might be the feed consumption. More specifically, this variable will be defined as the feed energy consumed over the duration of the feeding period. Energy is selected as opposed to total feed or some other standard because of its greater effect on gain as indicated by NRC (28). While there has been an increased use of net energy measurements in the recent past, the available data is recorded in total digestible nutrients, TDN. It is felt that the TDN units will accomplish the desired results and energy intake for the feeding period is, therefore, defined in terms of kilograms of TDN.

The next variable is obvious from the first two; both measure the variable over the duration of the feeding period. Therefore, some variable must specify the length of the feeding period or time. This variable will be defined as simply the days on feed. The days are computed from the first day on feed and thus include any warm-up period. This inclusion is based on the fact that different animals exhibit different reactions to warm-up feeding as in the case of compensatory gain. This difference in reaction can be accounted for in genetic variables.

Other factors which might also require attention include temperature and precipitation. These variables have been significant in other studies (15, 16). However, it is not the objective of this study to suggest optimal animals should vary greatly based on uncontrollable variables. For example, it is impossible for genetic selection to make changes as rapidly as the weather, therefore, selection must be based

on average weather conditions for some location. For this reason, weather variables will not be included in environmental parameters.

The next aspect must be the specification of genetic variables. In this regard, attention must be given to both the relevance to the feeder as well as limitations of the data. With these aspects in mind, several variables immediately come to light. These include average daily gain, feed conversion efficiency, breed of animal, type of animal, weight at the start of feeding, and weight at a fixed USDA slaughter grade.

The inclusion of average daily gain would be repetitive since time and gain are already included. The last two items concerning weight are also related to gain by subtracting initial weight from slaughter weight. Some information about these variables can be obtained, but inclusion as variables is perhaps not the most appropriate way to proceed. This leaves three variables to be evaluated: feed conversion efficiency, breed of animal, and type of animal. While the breed and type may initially seem to be identical, the variance between animals within a breed indicates this is not the case.

Breed of animal can easily be handled as a classification variable. In the context of the data, each of the nine crosses represents a breed classification. This treatment of breed has the effect of blocking on the breedlines so that the effect of breeding can be removed from gain.

Type of animal represents a more difficult problem. In practice, type of animal is usually a function of some structural aspect related to size. For example, producers refer to big-boned animals or just big or

small animals. These references to size are related to either the mature or final size of an animal or to an animal's present size, or both.

In the context of the data, an appropriate indication of type would be an estimate of an animal's mature weight. Since steers are slaughtered before maturity, the estimate must be made from another source. The basis for this estimate is the resemblance between relatives and since steers were selected such that they do not represent an isolated record on a sire, the relatives are the half-sisters which were maintained in the breeding herd.

In order to form this estimate of mature weight, an adjust growth curve relating age and weight is fit to the relevant cow data using least-squares regression, similar to that done by Brinks, et al. (1). The curves are fit such that an estimate of mature weight of progeny is obtained for each sire by breed of cow. Procedurally, this amounts to fitting nine growth curves, each including classification variables by sire. Significance levels for age and sire variables and R^2 for the curves are shown in Table 3.

In order to obtain mature weight estimates from the functions, a point is selected at which the slope of all functions is equal. Graphically, this is an equivalent to the tangency point on the curves to a line of given slope as shown in Figure 6.

These estimated mature weights by cross and sire are summarized in Tables 4 through 6. Since type is a relative or comparative value, the weight estimates can be related directly to the half-sib steers. In

Table 3. Estimated coefficient values, $\hat{\beta}$, for age, significance levels and R^2 for growth curves by line

Line Code	Sire ^a Sig.	Age		R^2 ^b
		$\hat{\beta}$	Sig.	
HH	.0013	65.2779	.0001	.7640
HA	.0001	66.3318	.0001	.8006
HS	.0005	64.4072	.0001	.8165
AH	.0001	65.5105	.0001	.7987
AA	.0001	58.6961	.0001	.8134
AS	.0001	57.7674	.0001	.8353
SH	.0001	63.5985	.0001	.7994
SA	.0023	60.1514	.0001	.8237
SS	.0043	56.5303	.0001	.7845

^aSire refers to the classification variable for individual sires.

^b R^2 is defined as the multiple correlation coefficient or

$$R^2 = \frac{\text{Sum of squares due to regression}}{\text{Sum of square total} - \text{Sum of square due to fitting intercept}}$$

other words, a larger cow than average indicates that a half-sib steer will be larger than average in general.

In keeping with reality, the producer is concerned not only with the mature size, but also with what the animal is currently. This relationship can be expressed as the ratio of current weight to mature weight, or degree of maturity. Mathematically, this relationship may be expressed as

$$DMT = \frac{WW}{MWT} \quad (29)$$

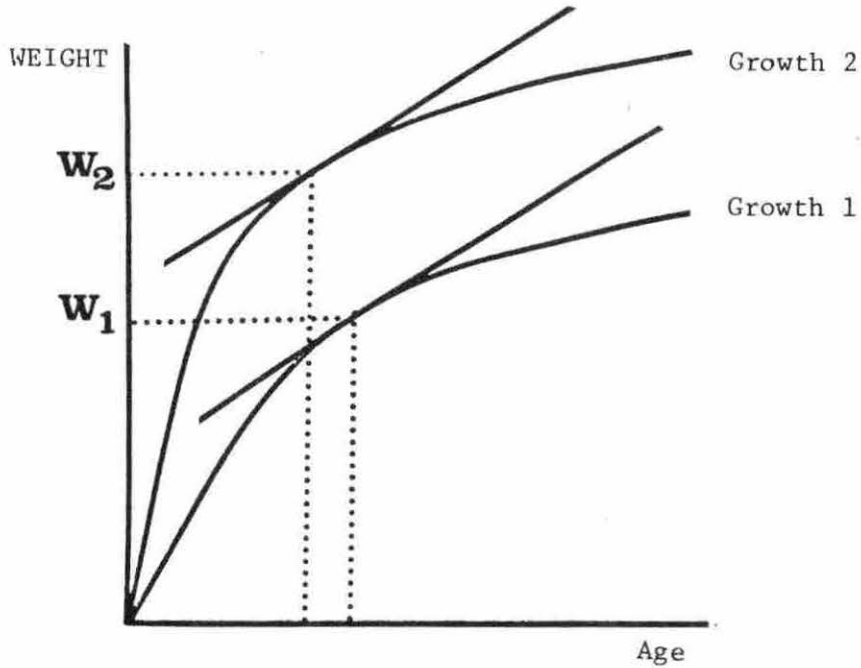


Figure 6. Illustration of weight differences of two growth curves at the same growth curve slope

where:

DMT = degree of maturity;

WW = weaning weight or weight at start of feeding trial in kilograms; and

MWT = estimated mature weight in kilograms.

In this manner, the variable DMT relates size at maturity as well as current size. This might be considered as choices between feeding calves or yearlings and between very large or very small types of cattle. In this context, an animal very close to final weight is different from one which is far from mature weight, even if they are at the same absolute weight.

Table 4. Mature weights of progeny produced by Hereford sires on Hereford, Angus, and Shorthorn cows--Growth Curve Slope = .10

Sire ID	Hereford kg. MWT	Angus kg. MWT	Shorthorn kg. MWT
57	388.97	397.05	
68	360.76	422.31	406.98
270	416.32	460.98	430.06
281	380.06		
345	372.29	420.42	477.39
347	404.74	433.73	
355	380.55		392.81
358	354.15	309.46	406.44
365	389.84	384.27	368.23
375	413.51	419.31	413.23
474	400.61	391.41	418.71
475	400.99	404.33	421.70
Mean	389.66	406.93	421.46
Δ Slope	.00015	.00015	.00016

Table 5. Mature weights of progeny produced by Angus sires on Hereford, Angus, and Shorthorn cows--Growth Curve Slope = .10

Sire ID	Hereford kg. MWT	Angus kg. MWT	Shorthorn kg. MWT
350	381.69	384.25	326.37
352	382.57	386.55	432.63
353		336.75	337.55
354	433.03	327.43	362.74
355			292.24
356	442.01	312.76	346.34
357	383.10	358.04	363.36
362		321.40	
481	381.91	350.37	358.20
483	382.53	353.73	343.21
484	377.34	348.05	377.34
486	398.75	371.46	378.02
637	395.73	371.75	
Mean	398.89	352.54	362.84
Δ Slope	.00015	.00017	.00017

Table 6. Mature weights of progeny produced by Shorthorn sires on Hereford, Angus, and Shorthorn cows--Growth Curve Slope = .10

Sire ID	Hereford kg. MWT	Angus kg. MWT	Shorthorn kg. MWT
348	391.90	368.29	331.55
349	386.58	371.35	345.37
350	383.05	360.39	358.72
351	427.01	357.98	393.25
477	399.99	399.63	372.99
478	430.49	394.40	354.06
479	355.48	375.21	341.03
480	352.51	344.90	335.33
986	388.54	373.66	306.65
987	394.77	322.04	321.07
988	364.26	355.05	333.28
Mean	391.42	365.42	346.73
Δ Slope	.00016	.00017	.00018

With this estimate of type as a variable, the remaining parameter is feed conversion efficiency. In general, feed efficiency is defined as the weight of feed consumed per unit of weight gain. Since all weight measures are in kilograms, and feed consumption is measured in TDN, the relationship can be expressed as

$$EFF = \frac{TDN}{Gain} \quad (30)$$

where EFF is the feed efficiency and TDN and Gain are as previously defined.

It should be obvious that feed efficiency is consistent with the properties of quantitative genetics in that it exhibits continuous variation. Furthermore, since it is not comparative as with type, and its observed value is phenotypic, additional computation is required to arrive at a breeding value for an individual.

The method for determining genotypic feed efficiency relies heavily on the theory of population genetics. The basic concept rests mainly with the principle of heritability, equation (14). The genotypic value of feed efficiency can be estimated by multiplying the variance in phenotypic value by the heritability of feed efficiency.

First, the assumption must be made that the expectation of environmental deviations is zero, i.e., $E(E) = 0$. Referring to equation (9), the result may be expressed such that the expectations of phenotypic value equals the expectation of genotypic value. The assumption that the expectation of environmental deviations is zero is not unusual

and usually realistic in experimental data where conditions are controlled as much as possible.

By writing equation (14) in expanded form the result becomes

$$G - \bar{G} = h^2(P - \bar{P}) \quad (31)$$

where:

\bar{G} = the expectation of genotypic value; and

\bar{P} = the expectation of phenotypic value.

By rearranging terms, the expression for genotypic value can be obtained as

$$G = \bar{G} + h^2(P - \bar{P}) \quad (32)$$

By noting that $\bar{G} = \bar{P}$, and using within breed estimates of heritability from published sources (5, 30), genotypic values of breed efficiency by individual can be obtained.

The selected variables have thus been defined and the production function to be estimated may be written implicitly as

$$g = f(E, T, B, EFG, DMT) \quad (33)$$

where:

g = gain in kilograms;

E = energy intake in kilograms of TDN;

T = time on feed in days;

B = breed effect--a classification variable;

EFG = genotypic value of feed efficiency; and

DMT = degree of maturity when the animal goes on feed.

With the variables defined, the properties of the relevant functions can be estimated and analyzed.

Empirical estimation

There are a variety of functional forms currently in widespread use in economics. Each of these functions exhibits some peculiarity and individuality in its properties. The determination of the appropriate form must initially be based on a comparison of the properties of a specific function to the biological properties involved. For example, there is a biological maximum feed consumption in steers, therefore, if gain results from consumption there must be a maximum gain. The resulting function should reflect that this maximum output exists.

In the case of genetic inputs, no such clear distinctions exist. For example, there is no theoretical or empirical evidence to suggest that genetic maximums exist, with evolution as a prime example. However, the biological limit on feed intake in a fixed genotype does exist. This property gives rise to a contradiction between genetic and environmental variables. There do, however, exist some definite relationships on which to base the initial selection of functional form.

One of the most obvious relationships shared by the inputs is the fact that each input is limiting. This relationship arises from the nature of inputs in a biological sense. Specifically, no gain is possible if energy, time, degree of maturity, or genetic feed efficiency is zero. In other words, an animal must have both some genetic and environmental level in order to produce.

This one aspect alone suggests that isoquants asymptotically approach the axis. Such isoquants are consistent with a Cobb-Douglas production function, which further indicates that there is no maximum output. This latter point is in keeping with the lack of theoretical maximums in relation to genetic inputs. In the case of two inputs, the Cobb-Douglas may be represented as

$$Y = X_1^\alpha X_2^\beta \quad (34)$$

An alternative form might be considered which does exhibit a maximum. A form most generally used in beef or animal functions is the quadratic form. This function may be expressed as

$$Y = aX_1 + bX_2 - cX_1^2 - dX_2^2 + eX_1X_2 \quad (35)$$

In this form, the function decreases due to negative signs on the squared terms.

Both functions are estimated using least-squares regression techniques. In both forms, since the data represent time series data, a problem of autocorrelation is evident. Autocorrelation arises from the violation of the least-squares assumption that the error terms are normally distributed and independent. Because of the time series nature of the data, the error in the t th period is related to the error in $t-1$ period, and not independent. This results in estimates which are not efficient. The transformation necessary to correct for the autocorrelation is similar to that done by Heady, et al. (15).

After correction for autocorrelation, a further problem arises. The estimated corrected parameters in both functions exhibit the opposite of the expected sign. Specifically, the functions estimated are everywhere increasing. Examination of least-squares estimation procedures indicates a possible reason for this occurrence.

The basic premise of least-squares is the minimization of the square error term, and hence, the name least-squares. Consider a linear function of the form

$$Y = \beta X + e \quad (36)$$

The estimates $\hat{\beta}$ then attempt to minimize $(e)^2$. In time-series data such as represented in this data set, a high number in one period is expected to be high in the next as well. What one desires is a fit which is the average of the animals, however, this may not be the case. This situation may be represented graphically in Figure 7.

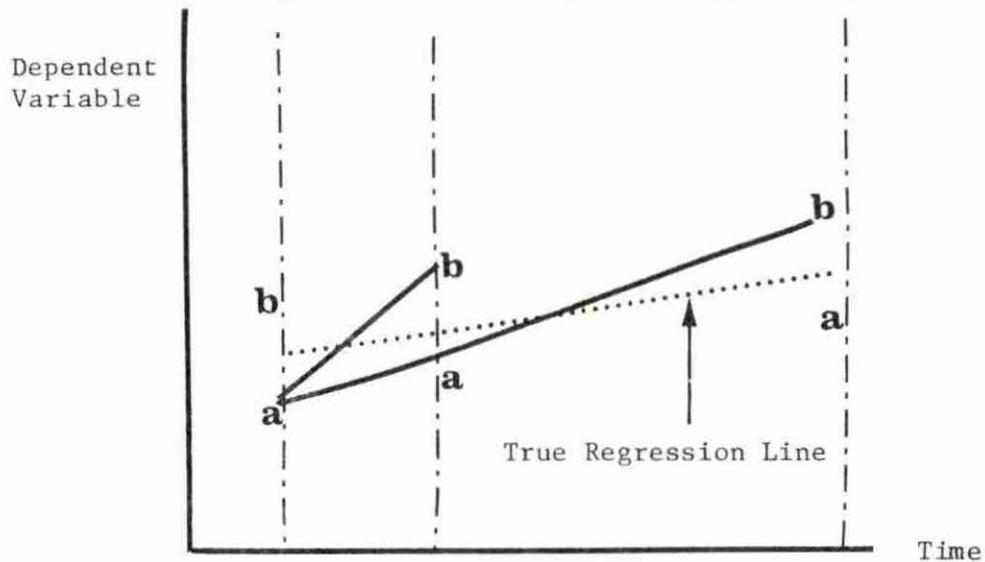


Figure 7. Impact of extending time period to reduce error of least-squares regression in time series data on two animals (a and b)

It can be seen that in this simple case the diagonal is the actual least-squares fit.

To compensate for this oddity in the data and procedure, the time-series nature of the data is abandoned. This has the result of reducing ten 28-day feeding periods to one period of 280 days. Since nine and ten period feeding trial were made, some variability in time is maintained. With this change, the data no longer requires adjustment for autocorrelation because there is now only one period.

In the case of the quadratic function, the empirical estimation results in a function of the form

$$\begin{aligned}
 g = & \tau_1 + 2.4737(E) - 13.4933(T) - 693.7499(EFG) \\
 & (.5094) \quad (.6677) \quad (.6025) \quad (.5564) \\
 & - 850.7776(DMT) - .0001(E^2) + .0053(T^2) + 3.0762(EFG^2) \\
 & \quad (.9056) \quad (.6533) \quad (.1843) \quad (.1124) \\
 & + 191.1779(DMT^2) - .0084(E \cdot T) + .0053(E \cdot EFG) - .0636(E \cdot DMT) \\
 & \quad (.2256) \quad (.7040) \quad (.7932) \quad (.7063) \\
 & + 2.5417(T \cdot EFG) + 2.3844(T \cdot DMT) + 8.5729(EFG \cdot DMT) \quad (36) \\
 & \quad (.5285) \quad (.9322) \quad (.8221)
 \end{aligned}$$

where τ_1 is the classification variable for breed. Significance levels as the probability of a greater t value are in parentheses below each variable. While this function results in a R^2 of .9928, none of the variables are significant at the .10 level and only two are significant at the .25 level.

The significance problems with a good fit are generally indications of multicollinearity in functions such as the quadratic. This multicollinearity is the results of the independent variables being highly

correlated with each other. In this case the linear, squared, and interaction terms are highly correlated and, therefore, the multicollinearity problem. Two methods are commonly used to break the multicollinearity deadlock: additional data or dropping terms in the function. As no additional data is available in this case, the alternative of dropping variables must be used. In order to reach acceptable significance levels on all variables, it is necessary to drop all squared terms and most interactions resulting in a basically linear function. Furthermore, the R^2 is reduced by this procedure to about .297. For these reasons, the quadratic function, equation (36), is abandoned.

The Cobb-Douglas estimation presents a much more satisfying result. The estimated function,

$$g = e^{\tau_i} E^{.74214} T^{.11117} EFG^{-.45813} DMT^{-.15269} \quad (37)$$

(.2411)(.0001)(.0994) (.0001) (.0031)

results in a R^2 of .9967. Significance levels, in parentheses below the variable, do not indicate multicollinearity. Also, all variables except the classification variable for breed, τ_i , are significant at the .10 level. These breed variables are estimated under the restriction that the sum of the effects is zero. The results allows independent appraisal of any one breed or the overall average and further suggests by the lack of significance that great differences between the given lines are absent.

The inclusion of an intercept in the estimation does not contribute significantly to the explanatory power of the function. Furthermore, the intercept is not significantly different from zero, with a

significance level .6685. This result is consistent with the premise that zero inputs yield zero output.

Properties of the function

With this estimated function, one is in a position to develop the relationships outlined under a positive response analysis. One item of special interest is again breed effects. Temporarily these effects will not be considered in the development of functional properties. This is equivalent to evaluating the function in terms of the average animal. Breed effects can later be developed as shifts from the average.

The initial relationships of interest are the marginal products of the inputs. These relationships

$$MP_E = .74214E^{-.25786} T^{.11117} EFG^{-.45813} DMT^{-.15269} \quad (38)$$

$$MP_T = .11117E^{.74214} T^{-.88883} EFG^{-.45813} DMT^{-.15269} \quad (39)$$

$$MP_{EFG} = -.45813E^{.74214} T^{.11117} EFG^{-1.45813} DMT^{-.15269} \quad (40)$$

$$MP_{DMT} = -.15269E^{.74214} T^{.11117} EFG^{-.45813} DMT^{-1.15269} \quad (41)$$

initially appear inconsistent with economic theory due to the negative signs on equations (40) and (41). However, examination in light of the definition of EFG and DMT indicates that the signs are correct. For example, an increase in EFG is identical to increasing feed consumption per kilogram of gain. By this definition, a larger number for EFG is worse than a smaller number and thus, the negative sign. By similar reasoning, this function implies in equation (41) that less mature

animals are more productive than more mature animals. This concept is also consistent with animal science theory.

These equations are consistent both from an economic and an animal science context. Furthermore, the equations also indicate that the marginal productivity of each input is dependent upon the level of usage of all inputs.

The next properties of interest are the elasticities of production. These values may be summarized as follows:

$$e_E = .74214$$

$$e_T = .11117$$

$$e_{EFG} = -.45813$$

$$e_{DMT} = -.15269.$$

Depending on viewpoint, these values represent either the strength or weakness of the Cobb-Douglas function. Since the values are constants, they imply that a percentage increase in an input's use will yield a constant percentage increase in output. Genetically this property is reasonable, but from the standpoint of feed intake it may not be.

Another property is the scale effect, r , which is the sum of the elasticities of production. In the estimated function, the scale effect is equal to .24249, which is less than one as required previously.

A final aspect of the function which will be developed is the marginal rate of substitution. As outlined, this property is one of the most significant. The marginal rates of substitution for the four inputs can be expressed as follows:

$$\begin{aligned} \text{MRS}_{E,T} &= .1498 \left(\frac{E}{T} \right) \\ \text{MRS}_{E,EFG} &= -.6173 \left(\frac{E}{EFG} \right) \\ \text{MRS}_{E,DMT} &= -.2057 \left(\frac{E}{DMT} \right) \\ \text{MRS}_{T,EFG} &= -4.1210 \left(\frac{T}{EFG} \right) \\ \text{MRS}_{T,DMT} &= -1.3735 \left(\frac{T}{DMT} \right) \\ \text{MRS}_{EFG,DMT} &= .3333 \left(\frac{EFG}{DMT} \right) \end{aligned}$$

As with the formulas for marginal product, equations (38) through (41), marginal rates of substitution involving a genetic and environmental variable are negative.

From both the marginal productivities, and the marginal rates of substitution, it is obvious that the value of these relationships hinges on the level of the inputs and are, therefore, not constant. Because of the number of possibilities, sample values are deferred until Appendix A rather than being presented at this time.

The remaining property is the breed effect. As noted, these effects may be considered as shifts in the production function. The values of breed effect are summarized by line in Table 7.

Table 7. Coefficient of breed effects by line

LINE	HH	HA	HS	AH	AA	AS	SH	SA	SS
EFFECT	-.0138	.0306	-.0278	-.0010	-.0271	.0101	-.0206	.0279	.0217

By considering all variables except E, the TDN consumption, as fixed, the breed effects can be depicted graphically, as in Figure 8. For example, the breed line HA, has the greatest positive value in Table 7 and represents the highest curve in Figure 8.

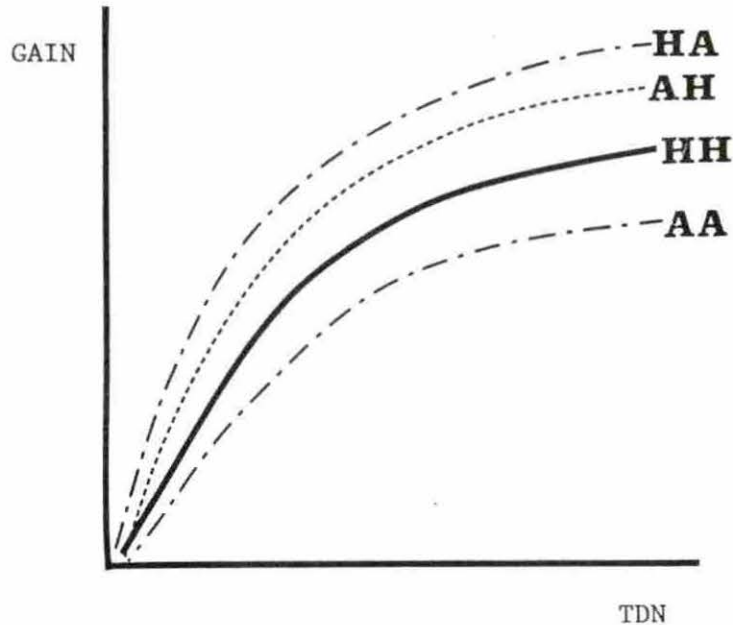


Figure 8. Relative production functions by breed line with all inputs except TDN considered fixed

The crosses involving Shorthorn have been left out of this graph. This omission is based on the fact that currently so few Shorthorns are available relative to Herefords and Angus, that they do not represent a viable selection alternative. The omission will also apply to subsequent analysis, but the overall mean will include the Shorthorn cross effects.

It can be seen from Figure 8 that for any given level of gain, the higher curves, HA and AH, require less feed. This would imply that the

crossbreds are better than purebreds, AA and HH, with all other factors fixed. This result is consistent with the explanation of heterosis or hybrid vigor presented previously. The same result can be seen in cost terms by allowing energy and time both to vary and graphing the resulting isoquants for a fixed gain. This procedure is presented in Figure 9 for 200 kilograms of gain. The breed line with the lowest isoquant represents the least cost manner of producing the desired gain with all other variables fixed.

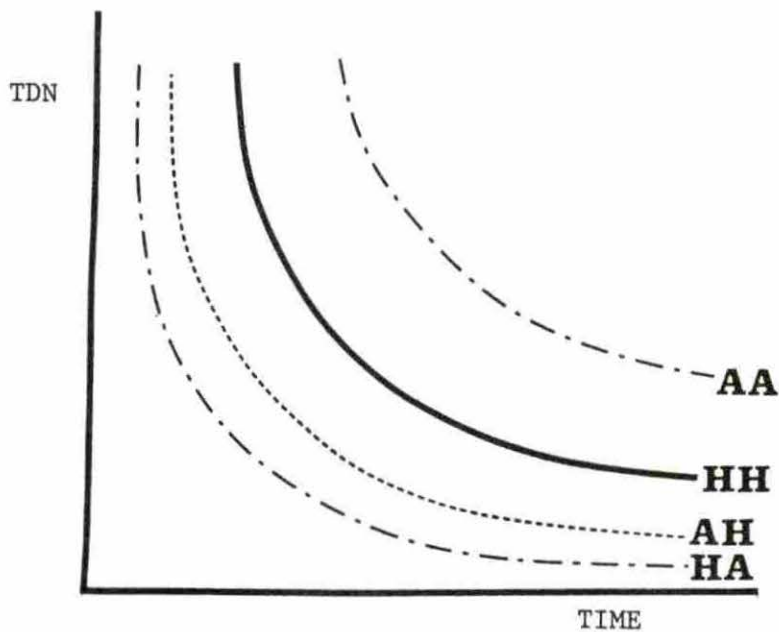


Figure 9. Typical isoquants by breed line for 200 kilograms gain and genetic variables fixed

Both Figure 8 and Figure 9 suggest that the Hereford bull on an Angus cow is the optimal cross. While this may be true, one of the assumptions made in the development of the graphs will cause this conclusion to be violated.

The assumption in question is the one concerning fixation of variables across lines. What this implies is that if all lines have the same level of genetic values, EFG and DMT, then the result holds. However, there is no reason to believe that this is the case. If each line has a genetic level peculiar to it, then the results may be entirely different.

Examination of the data reveals that there are distinct differences in genetic values as outline in Table 8.

Table 8. Means and standard deviations, σ , of genetic variables by breed line

Line Code	EFG Mean	σ	DMT Mean	σ
HH	3.5984	.3733	.5106	.0606
HA	4.0006	.5442	.5311	.1056
AH	3.6957	.5947	.5185	.0768
AA	4.2893	.4434	.6092	.0768
Overall ^a	4.0921	.9216	.5610	.0862

^aOverall includes Shorthorn crosses.

The Linear Programming Analysis

The differences between breed lines indicated in Table 8 suggest that the assumption of fixed genetic values may be unrealistic in this data set. To further analyze these differences and their implications on profit, a linear programming model as outlined previously is required.

Development of the model

The basic premise in the development of this model is that the producer can distinguish between the breed lines. Given some breed, and genetic level within a breed, he then determines how much gain is desired and how much time and energy to use. This is equivalent to selecting a single optimal point on the production surface of each breed group.

In the context of the model, the conditions outlined are achieved through a variety of restrictions and assumptions. First, since the initial distinction is between breed lines, a model is formulated for each line. Given that the structure of each model is the same, the only differences arise from such differences as those shown in Table 8. Since the operator can be assumed to maximize profit, comparison of the values of the objective function should indicate the most profitable breed choice. Therefore, the initial aspect of the model is that it represents a breed line and secondly, the model is formulated with a profit maximizing objective function.

Next, consideration must be given to the appropriate handling of the production function in the context of the linear programming model. Approximation of the function by linear segments indicates that five

gain levels will form a reasonable estimate. To these five levels, the mean gain is added for additional proximity to reality.

Because of the lack of ability to vary genetic level after the selection of the animal, these inputs are initially considered fixed. In this manner, energy/time isoquants are derived and linearized. This procedure indicates that the isoquants may be approximated in six segments at low and intermediate levels of gain and by five segments at high levels of gain.

The linear approximations of gain, time, and energy relationships indicate the use of approximately 36 activities in the model thus far. In this context each activity represents a point on the production surface or a specific combination of gain, time and energy. If two of the six levels of gain can be considered high, then the number of activities is reduced by two to thirty-four.

Thus far, nothing has really been said about the genetic variables except to initially consider them as fixed. Since the objective of the study concerns selection, it is reasonable to initially approach them from this viewpoint. Since selection deals with improvements in the herd average, it is necessary to provide activities representing the herd average and better. If three levels of genetic feed efficiency and two levels of degree of maturity are selected, then the model has 170 to 180 activities, or five for each gain, time and energy level. However, due to the genetic correlations, it is unnecessary to evaluate both traits simultaneously at improved levels. Removing these activities results in 136 to 144 individual points on the production surface to be evaluated for each breed line.

The activities remaining represent consideration of six levels of gain. Each level of gain may be produced by six levels of energy and time for each of four combinations of genetic values, or twenty-four ways to produce one level of gain.

In an attempt to interpret the inputs, it can be seen that the producer is generally free to vary both time and energy during feeding. In other words, once an animal is selected for feeding, the genetic level is fixed and only time and energy are variable. This condition is incorporated into the model by including activities which supply energy and time to the model as needed at some cost. The genetic level, on the other hand, is restricted such that there is a fixed level from which to draw for production.

The model is further restricted to selecting one activity and thus, only one point on the production surface. In this manner, the production function model contains five rows, not including the objective and 138 activities including one supply activity each for time and energy.

These conditions may be represented more graphically by the initial tableau in Figure 10. This table represents the same conditions in a reduced model consisting of two gain levels, two energy and time combinations and two levels of each genetic input.

Mathematically, this model may be represented as

$$\text{Max } Z = \sum_j C_j X_j \quad (42)$$

subject to

$$\sum_j a_{ij} X_{ij} = 1 \quad i = 1 \quad (43)$$

	GAIN 1				GAIN 2				RHS			
	Genetic Level 1		Genetic Level 2		Genetic Level 1		Genetic Level 2					
	Time 1	Time 2	Time 1	Time 2	Time 1	Time 2	Time 1	Time 2				
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8			$-C_9$	$-C_{10}$
OBJ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	$-C_9$	$-C_{10}$		
SEL	1	1	1	1	1	1	1	1			=	1
TDN	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	-1		\leq	0
TIME	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}		-1	\leq	0
EFG	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}			\geq	b_4
DMT	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}			\geq	b_5

Figure 10. Sample linear programming initial tableau for reduced production model

$$\sum_j a_{ij} X_j \leq 0 \quad i = 2, 3 \quad (44)$$

$$\sum_j a_{ij} X_{ij} \geq b_i \quad i = 4, 5 \quad (45)$$

where:

Z = value of objective function;

C_j = value of one unit of j th activity to Z ;

X_j = number of units of j th activity;

a_{ij} = technical coefficient indicating the use of the i th input
in the j th activity; and

b_i = resource limit on i th input.

In this abbreviation model, equation (43) is the selection restraint requiring selection of one activity. Formula (44) represents the energy and time use rows and formula (45) is a representation of genetic restrictions. The genetic restrictions are greater than or equal to b_i rather than the expected less than because a lower numeric value for the traits is better than a higher value as specified by the definition.

The coefficients

The coefficients for all genetic and environmental variables are derived from the production function. It has been shown that both the production function and the isoquants are approximated by linear segments. The coefficients represent the endpoints of these line segments, and are estimated by successively considering fixed levels of time and genetic inputs and solving for TDN at the given gain level.

To better appreciate the coefficient estimation procedure, consider estimation of the coefficients for a gain of 200 kilograms in Hereford steers (HH). The first set of coefficients are derived with the genetic inputs, EFG and DMT, set at the line means, as shown in Table 8. Next, time is varied from 60 to 240 days in 60-day intervals and the isoquant is solved for TDN at each time level. This estimation is accomplished using only the relationships of the estimated production functions.

This relationship for TDN required for 60 days on feed may be expressed as

$$\text{TDN} = \left(e^{-.01378} 60^{.11117} 3.5984^{-.45813} .5106^{-.15269} \right)^{-1.34745} 200^{1.34745} \quad (46)$$

Equation (46) is solved for each of the six time levels at the mean genetic levels. Then, the same procedure is repeated as EFG is improved by one and two standard deviations with DMT at the line mean. Next, EFG is held at the line mean and DMT is improved by a standard deviation. This results in 24 points estimated on each gain level.

In terms of the production function, these points represent an isoquant schedule, or points on a specific gain isoquant. These isoquant schedules and thereby, the coefficients of the relevant linear programming model are included in Appendix A.

The selection restriction represents no problem. It simply insures that the optimal solution to the linear programming model will include exactly one point on the production surface. In other words, the optimal solution will include one level of gain and one combination of inputs.

The coefficients of the objective function, the C_j , represent a difficult problem to solve. It has been previously shown that the feedlot operator is involved in producing gain, yet no market actually exists for gain. Since the objective function values for all activities, with the exception of those supplying energy and time, are the returns received for gain, this is a substantial problem.

The method used to solve this problem finds its basis in financial theory. It is assumed that the producer requires an amount for gain sufficient to cover his opportunity cost for capital and the cost of maintaining the animal. This assumption may be more fully illustrated by referring to a feedlot situation. The producer purchases a feeder steer at the market price, and has this purchase amount tied up until the steer is sold. Since the money is not available for other uses, it has a cost equal to the best foregone use, or opportunity cost. This cost must be somewhat subjective, but it is reasonable to assume that the feeders return exceed interest costs on feeder cattle. Based on this assumption, the opportunity cost is subjectively set at an amount equal to the feeder cost compounded at 10 percent for the duration of the feeding period. In other words, the feeder requires a 10 percent compound rate of return on his investment in feeder cattle.

The next point is the maintenance of the feeder steer. Before any gain can be realized, the initial weight must be maintained. This maintenance requirement is based on the weight of the steer and supplied from the feed consumed. The requirement for maintenance may be determined on a daily basis by using the formula

$$E = .077 W^{.75} \quad (47)$$

where:

E = energy in megacalories per kilogram of net energy per day, and

W = weight of the feeder;

as presented by the NRC (28). By using the same source, it is also possible to determine the maintenance energy contribution of various feed-stuffs. The cost of maintenance can be determined based on the market cost of the feed and the days on feed. Since corn is the accepted standard, it will be used as the feed in this estimation, with values from NRC tables (28).

Based on these two aspects, the value of gain or return can be expressed as

$$P_g g = P_s W_s - P_f W_f (1 + i)^t - M_t \quad (48)$$

where:

P_g = price of gain per unit weight;

g = amount of gain;

P_s = price of slaughter steers per unit weight;

W_s = weight of slaughter steers;

P_f = price of feeder steers per unit weight;

W_f = weight of feeder steers;

i = compound interest rate = 10 percent per year = .0278 percent per day;

t = days on feed; and

m = maintenance cost of feed per day.

It should be noted that this formulation assumes that the producer is not willing to invest labor or capital without gain, therefore, maintenance costs are totally represented by feed costs. Furthermore, the same uncertainty about slaughter prices faced by producers persists.

To form an expectation of slaughter price, season variations are assumed to be zero. This eliminates price fluctuations resulting in capital gains. Examination of market data over the period from August 1974 to December 1975 indicates that slaughter prices are higher than feeder prices by some proportional relationship, i.e., the price lines are parallel (38). Specifically, the price of 900-1,300 pound slaughter steers is an average of 1.3 times the price of 500-700 pound feeder steers, with average feeder prices of \$.7379 per kilogram or \$.335 per pound.

The final problem is determining the price for an animal which may not fall into either category. This problem is overcome by translating the weights into degrees of maturity based on the maturity weight of the data set used to estimate the production function. In this manner, feeders are defined as .55 mature and slaughters as 1.1 mature, for a change of .55 in maturity as the market standard. Since this change in maturity is directly associated with change in price, the expected price of any animal can be estimated based on the relative change in maturity times the standard price change of 1.3. This relationship may be depicted as

$$P_s = P_f X \alpha \quad (49)$$

where:

$X = 1.3$, the standard price change when change in maturity is .55;

α = relative change in maturity or the actual change divided by .55.

Graphically, this relationship is represented by Figure 11. In this illustration, feeder price, F is associated with some maturity over time as is slaughter price, S. The producer might buy at point A and move along the diagonal to point B. By this formulation, intermediate points can be priced based on the relative distance covered. It should be noted that such formulation may hold only between and in the immediate vicinity of the price lines because of such things as price penalties on overfattened or overmature animals.

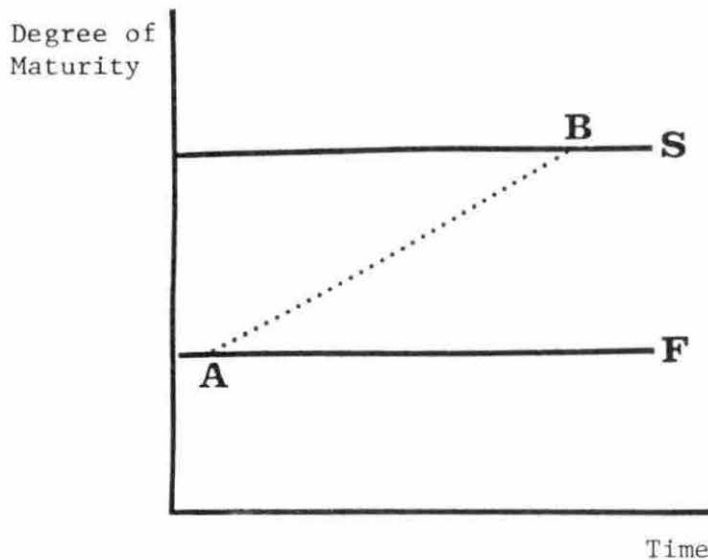


Figure 11. Price and maturity relationship over time

By combining equations (48) and (49) the value or return of gain, $P_g g$ can be expressed as

$$P_g g = P_f X\alpha G + P_f W_f (X\alpha - (1 + i)^t) - Mt \quad (50)$$

This formulation is then used to determine the objective function values for the various gain activities.

A nonrigorous examination of this function indicates conformity with economic theory. Specifically, dividing equation (50) through by P_g results in an inverse relationship between gain (g) and price of gain P_g , or a downward sloping demand for gain. Additionally, the amount of gain demanded is inversely related to both maintenance requirements and opportunity costs of capital. This is consistent with intuition suggesting less gain will be demanded as maintenance, time, interest rates, etc., rise. Also, gain demanded increases as it contributes to $X\alpha$, or actually to the grade of the animal.

The final coefficients needed are those supplying energy and time to the ration. Each of these activities supplies a kilogram of TDN and a day on feed is based on labor costs of about two cents per day and medical, fuel and supplement costs of about six cents per day for a total of eight cents per day. TDN costs are based on the cost of supplying one kilogram of corn grain TDN.

To fully analyze the feedlot situation, it is necessary to explore more than the simple price structure thus presented. For example, this price structure illustrates a constant margin based on feed prices. To

correct for this situation and account for drastic price fluctuations facing producers, prices are varied. Specifically, time cost is considered invariant, but corn prices vary from \$1.50 per bushel to \$3.50 per bushel in \$1.00 increments. Since this feed price also effects the value of gain through maintenance cost, increasing corn prices have a double effect on reducing the producer's margin. This treatment results in solving each model under high, medium, and low price margins.

The final element necessary for completion of the model is the restriction level, or the b_i . These restrictions represent the genetic levels available to the producer. As previously outlined, the activities represent feeding at various levels of genetic ability, however, with any animal there is only one genetic level available. This situation is represented in the model by restricting the genetic level at each of the four levels mentioned. The result is four solutions of the model; one for each genetic level. These levels are determined by the breed lines and, therefore, reflect line differences as well as within-line differences. With three price sets and four restriction levels, there are twelve solutions to each breed line model.

In order to complete the solutions, one final restriction is made. This restriction is that the producer must sell finished animals at the specified degree of maturity, 1.1. This restriction is based on a current lack of market for either more or less finished animals than the standard. This aspect is built into the model by varying weaning weights and mature weights so that DMT is at the line mean for all levels of gain. In other words, an animal which gains 325 kilograms has both a larger mature

weight and weaning weight than one gaining 150 kilograms even though both have the same degree of maturity on feed, DMT.

Linear programming solutions

Since a price structure would be common to all breeds at a point in time, the most appropriate evaluation would be that based on price margins. In this regard, all breed lines are compared under high, medium and low margins at mean genetic levels and improved levels.

Under low feed prices Initially, under high price margins or low feed prices, an analogy to the situation facing producers 10 years ago can be made. Corn is priced at \$1.50 per bushel and the producer has adequate margins across all breeds. The linear programming selection of the most profitable feeding system by genetic levels can be seen in Tables 9 through 12.

Table 9, which presents the situation where genetic variables are at the line means, is initially of the greatest interest. While these means represent data from 10 years ago, if the breeds made proportional progress in selection for the genetic variables, the same relative position holds now. Examination of the value of gain for each line indicates that the profit maximizing producer would select the average Hereford steer, line HH. Also, in this line, as well as all other lines except Angus, line AA, he would select animals with larger mature weights. He could then put the most gain possible on these animals, given the marketing restriction on maturity.

Table 9. Optimal solution by line under low feed prices (high margins) with genetic variables at line mean

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.5983	.5106	180	2178.26	325	197.28
HA	4.0007	.5311	180	2192.43	325	177.48
AH	3.6957	.5185	180	2183.55	325	189.30
AA	4.2893	.6092	120	1219.52	250	67.68
Overall	4.0920	.5610	180	2360.06	325	135.35

Table 10. Optimal solution by line under low feed prices (high margins) with DMT at line mean and EFG at line mean plus one standard deviation

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.2250	.5106	180	2035.87	325	208.04
HA	3.4565	.5311	180	2003.22	325	191.78
AH	3.1010	.5185	180	1959.41	325	206.24
AA	3.8459	.6092	120	1787.94	250	77.10
Overall	3.1660	.5610	180	2014.34	325	161.49

Table 11. Optimal solution by line under low feed prices (high margins) with DMT at line mean and EFG at line mean plus two standard deviations

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	2.8518	.5106	180	1886.99	325	219.30
HA	2.9123	.5311	180	1797.15	325	207.37
AH	2.5063	.5185	180	1718.09	325	224.49
AA	3.4025	.6092	180	2221.60	325	89.84
Overall	2.2398	.5610	180	1629.61	325	190.78

Table 12. Optimal solution by line under low feed prices (high margins) with EFG at line mean and DMT at line mean plus one standard deviation

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.5983	.4500	180	2122.34	325	320.43
HA	4.0007	.4211	180	2090.19	325	234.04
AH	3.6957	.4385	180	2109.54	325	281.59
AA	4.2893	.5292	180	2489.88	325	361.22
Overall	4.0920	.4748	180	2280.44	325	389.06

As noted, Angus steers are the exception. If August steers are fed, the profit maximizing producer would select steers of an intermediate size to gain 250 kilograms. This situation is due to the high degree of maturity exhibited by these steers, .6092 as compared to an overall average of .5610.

Tables 10 through 12 present the impact of nonproportional changes in the genetic variables. Specifically, the variables are changed by the variables' within-line standard deviations.

Table 10, when EFG is improved by one standard deviation, represents no change of significance. However, in Table 11 where EFG is improved by two standard deviations, the Angus and Hereford cross, AH, is the overall optimal. All animals have, of course, become more profitable, but the AH has become proportionally better. One additional change is that when EFG in Angus steers is increased by two standard deviations, the optimal policy is selection of large animals as in the other lines.

Table 12, indicating the impact of one standard deviation change in DMT, shows a substantial shift in production so that Angus steers are optimal. This indicates that from the producers' viewpoint an Angus calf with a large mature size and small weaning weight, or weight on feed is preferred. The shift in ranking can be explained by a greater effect on Angus of lower DMT than on other breed lines.

While Tables 9 through 12 answer the general question of which animal, the value of the genetic traits has not been determined. As outlined previously, this value or imputed price, will be determined by the shadow prices in the optimal solution. This value is invariant with

respect to changing genetic levels, but does vary with breed lines. The values of the genetic inputs are presented by breed in Table 13.

Table 13. Value of genetic variables by breed line under high price margins

	HH	HA	AH	AA	Overall
EFG	29.49	27.46	29.59	24.98	29.92
DMT	2032.14	514.18	1153.57	3669.27	2943.38

The values in Table 13 represent the value of a full unit of the genetic variables. For example, improving EFG in breed HH from 4.0 to 3.0 is worth \$29.49. However, the value for DMT is exaggerated since DMT is defined as greater than zero. The adjustment for this point simply requires reduction from one unit to a smaller unit such as one-tenth, or one-hundredth. In this manner, the value of improving DMT from .5 to .4 in breed HH is \$203.214.

One interesting aspect of these values is that the crossbreds have a lower value for DMT than do the purebreds, HH and AA. The implications of these values is quite simply that under high price margins there is less benefit, if any, to be gained in either purebreds, or the overall mean. This value for the genetic traits, whether it is EFG or DMT, does not have to be marketed independently, as the value is derived from feed savings and time savings during feeding.

Under medium feed prices The next situation considered is that of intermediate price margins corresponding to \$2.50 per bushel corn grain. This is comparable to the situation feedlot operators are currently facing.

Optimal production strategies under the medium price margin situation are reflected in Tables 14 through 17. It can be seen from examining the breed mean inputs that the Hereford steers are superior once again. All values are significantly lower, to the point that the best that can be expected with Angus steers, AA, is a loss of \$8.36.

As significant as the reduction in profitability, is the change in production state. Under situations of low feed prices and high margins, all but the Angus steers, AA, are selected at the largest possible and all possible gain made. Under lower price margins, the animals selected as profit maximizing are smaller and the gain is reduced by 100 kilograms to 150 kilograms. This is the case in all breed lines except the Angus, AA, and the overall, where the smallest possible animals for minimum gain are selected.

Tables 15 through 17 follow the same pattern as Tables 10 through 12 except that the Herefords, HH, are less than best with only one standard deviation improvement in EFG. One other point of interest is that two standard deviations improvement in EFG is required before even a positive return on the Angus steers is possible.

The value of the genetic inputs under an intermediate price margin are shown in Table 18. These values are quite similar to those of Table 13. One very noticeable point is that the values of EFG and DMT in

Table 14. Optimal solution by line under medium feed prices (medium margins) with genetic variables at line mean

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.5983	.5106	120	1625.39	250	72.15
HA	4.0007	.5311	120	1635.95	250	58.11
AH	3.6957	.5185	120	1629.33	250	67.89
AA	4.2893	.6092	120	960.89	150	-8.36
Overall	4.0920	.5610	120	884.78	150	27.60

Table 15. Optimal solution by line under medium feed prices (medium margins) with DMT at line mean and EFG at line mean plus one standard deviation

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.2250	.5106	120	1519.14	250	85.53
HA	3.4565	.5311	120	1494.77	250	75.90
AH	3.1010	.5185	120	1462.08	250	88.97
AA	3.8459	.6092	120	898.30	150	-.48
Overall	3.1660	.5610	120	1503.07	250	55.99

Table 16. Optimal solution by line under medium feed prices (medium margins) with DMT at line mean and EFG at line mean plus two standard deviations

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	2.8518	.5106	180	1886.99	325	102.10
HA	2.9123	.5311	180	1797.15	325	97.11
AH	2.5063	.5185	180	1718.09	325	118.94
AA	3.4025	.6092	120	832.87	150	7.77
Overall	2.2398	.5610	120	1213.97	250	88.46

Table 17. Optimal solution by line under medium feed prices (medium margins) with EFG at line mean and DMT at line mean plus one standard deviation

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.5983	.4500	180	2122.34	325	198.67
HA	4.0007	.4211	180	2090.19	325	112.98
AH	3.6957	.4385	180	2109.54	325	160.51
AA	4.2893	.5292	180	2489.88	325	220.08
Overall	4.0920	.4748	180	2280.44	325	258.65

Angus steers are substantially less under the intermediate margin than under high price margins. This would indicate that there is less benefit to be gained from improvement in traits in this case than under high margins. This peculiarity can be at least partially explained by comparison of the optimal Angus feeding program in Table 14 to that in Table 9. There is no difference in the number of days used; the cost of which does not vary from a gain of 250 kilograms (Table 9) to 150 kilograms (Table 14). This is the only breed which exhibits this situation. It is, therefore, possible that at least a portion of the increased values on all lines except Angus is due to a reduction in the number of days on feed.

Table 18. Value of genetic variables by breed line under medium price margins

	HH	HA	AH	AA	Overall
EFG	40.11	35.83	42.92	18.19	32.86
DMT	2087.70	498.78	1157.67	2855.47	2680.47

Under high feed prices The obvious final step in the linear programming analysis is the situation of low margins brought on by high feed prices. This situation is comparable to the position in which many producers found themselves during 1973 when prices for feed grains soared to record highs.

The profit maximizing strategies are shown in Tables 19 through 22. It can be seen that the Hereford steer is still the best the producer can

Table 19. Optimal solution by line under high feed prices (low margins) with genetic variables at line mean

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.5983	.5106	120	816.59	150	9.93
HA	4.0007	.5311	120	821.94	150	-2.31
AH	3.6957	.5185	120	818.61	150	6.15
AA	4.2893	.6092	120	960.89	150	-66.15
Overall	4.0920	.5610	120	884.78	150	-25.00

Table 20. Optimal solution by line under high feed prices (low margins) with DMT at line mean and EFG at line mean plus one standard deviation

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.2250	.5106	120	763.22	150	19.34
HA	3.4565	.5311	120	751.00	150	10.20
AH	3.1010	.5185	120	734.58	150	20.97
AA	3.8459	.6092	120	898.30	150	-55.11
Overall	3.1660	.5610	120	755.18	150	-2.13

Table 21. Optimal solution by line under high feed prices (low margins) with DMT at line mean and EFG at line mean plus two standard deviations

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	2.8518	.5106	120	707.41	150	29.19
HA	2.9123	.5311	120	992.77	200	23.68
AH	2.5063	.5185	120	949.09	200	38.89
AA	3.4025	.6092	120	832.87	150	-43.57
Overall	2.2398	.5610	120	609.93	150	23.49

Table 22. Optimal solution by line under high feed prices (low margins) with EFG at line mean and DMT at line mean plus one standard deviation

Line Code	EFG Level	DMT Level	Time in Days	TDN kg.	Gain kg.	Value of Gain
HH	3.5983	.4500	180	2122.34	325	76.95
HA	4.0007	.4211	120	821.94	150	-2.31
AH	3.6958	.4385	180	2109.54	325	39.01
AA	4.2893	.5292	180	2489.88	325	78.92
Overall	4.0920	.4748	180	2280.44	325	128.24

do, but even at that he can expect a return of less than \$10. The average Angus steer, which is the worst choice, would be expected to lose over \$65.

The results are quite similar to those of medium margins, except once again smaller animals are selected, and less gain is marketed; 150 kilograms in Table 19 as opposed to 250 kilograms in Table 14. Furthermore, the small animal is preferred until two standard deviations improvement in EFG are brought about as in Table 21. Even with this substantial improvement, not all breeds increase and those that do increase only by 50 kilograms of gain.

One other point of interest is that Angus steers never reach a positive return even with improved EFG levels. Only when DMT levels are improved does it become profitable to feed Angus steers under low margins.

The values of DMT and EFG as shown in Table 23 indicate generally less benefit in traits than in Table 18. Specifically, under low margins, there is no value to improved DMT in the HA steers. This would indicate that this line is as large as is feasible under these restrictions. This implication is derived from the manner in which DMT may be changed; either mature size or weaning size. Since weaning size is closely related to a dam's ability, mature size can be considered as the predominant genetic force.

In order to form some general decision rules from these various results, trends across the various margins are examined. The initial and most obvious of these trends is that as price margins decline producer's return also drops. Combined with falling returns is a fall in

production and animal size. As the producer attempts to compensate for decreased margins he markets less gain on the animals. In order to put on less gain and still market animals at the accepted maturity, he feeds smaller animals.

Table 23. Value of genetic variables by breed line under low price margins

	HH	HA	AH	AA	Overall
EFG	25.80	23.88	27.53	25.47	26.18
DMT	1105.87	0	410.75	1813.45	1777.67

An additional point of interest is that DMT generally has a much larger effect than EFG. This indicates that the initial part of gain is important to producers and that if possible less mature animals should be fed. This corresponds to feeding calves from weaning as opposed to yearlings, unless those yearlings are small for their age. This is the situation of yearlings which have low degree of maturity for their age and, therefore, exhibit compensatory gain.

CHAPTER IV. SUMMARY

This study presents an alternative method by which economic principles and animal breeding concepts can be combined in a single model. While the study is confined to a feedlot situation in the analysis, the principles can easily be extended to any phase of beef production. Additionally, minor revision of certain assumptions permits adaptation of the same model to any industry utilizing genetic inputs.

The process of combining these theories rests on the economic theory of the firm, which initially requires estimation of a production function. The most appropriate functional form estimated is the Cobb-Douglas, blocked by breed lines, as shown in equation (37). The analysis of this function indicates that the greatest efficiency in production can be achieved using the crossbred steers as opposed to purebreds. This finding is consistent with heterosis theory in genetics, but is contingent on identical genetic levels. In other words, if the only difference between two animals is the breeding, then the crossbred is superior. Within the four lines examined, the use of Hereford bulls on Angus cows is the most efficient cross and the straight Angus is the least efficient. It should again be stressed that this situation exists only when all lines are evaluated at the same genetic level.

When there is no presumption that the genetic level is identical between lines, a linear programming model can be used to evaluate both breeds and genetic inputs. This model is formulated such that the genetic

level of a line is determined by that line's mean and standard deviation for the given trait. In this manner, both the lines and traits can be evaluated from the standpoint of a profit maximizing producer. The use of linear programming in no way excludes the estimated production function, because the technical coefficients of the linear program are derived from the production function.

The results of the linear programming analysis indicate that the straight Hereford is the profit maximizing selection across low, medium, or high price margins. This result does not contradict the relative efficiency of crossbreds mentioned earlier, but simply indicates improved genetic levels in Hereford steers in the data.

A strong result of the analysis is that of optimal gain levels. Given a pricing of gain which reflects the opportunity cost of capital invested in the feeder steer and maintenance cost for the feeder, the optimal amount of gain falls as prices of feed increase. This result further implies that the mature size of the optimal animal falls as well, due to the restriction that the slaughter animal sold must conform to grading and maturity standards. This result would appear to be in direct conflict with an industry trend toward the large exotic breeds. The only justification for the use of such animals in times of high feed prices must be a substantially higher genetic level. Perhaps a more reasonable strategy would be the improvement of genetic feed efficiency in domestic cattle of smaller size, such as Hereford and Angus.

Furthermore, if the data is representative of current industry standards, recommendations to feedlot operators can be formed. First, as prices for feed increase, the mature size of the animal fed should decrease. Second, calves are preferable to yearlings in general due to a lower degree of maturity, DMT; and third, the choice of breed rests on the genetic ability of the breed. If there is no reason to suspect a difference in ability, the crossbred steers, especially HA are optimal. However, if genetic levels differ comparable to that in the data used, selection of optimal must be based on these genetic levels rather than only breed differences.

Based on a knowledge of the price and size relationship, as well as the genetic levels of the lines considered, the producer can evaluate his selection of an animal to feed. There still remains the question of value for the two traits analyzed, genetic feed efficiency and degree of maturity. The linear programming analysis also sheds light on this question.

The shadow prices, or marginal value products, of the traits in the linear programming analysis may be interpreted simply as their value. This value does not require marketing, as it is achieved through a cost saving to the producer. These values do, however, suggest the amount a feedlot operator would be willing to pay the cow-calf producer for the added genetic ability.

In general, the values of genetic traits are fairly uniform across lines. The most notable exception is the Angus steer, which exhibits values for EFG which are substantially lower under medium price margins

and an exceptionally high value for DMT under high margins. The other lines exhibit a cyclical type reaction to feed price changes with respect to EFG so that EFG value is higher under medium margins than any other. DMT on the other hand exhibits a generally falling value as feed prices rise.

The reaction of DMT value to feed price changes is consistent with the change in optimal animal and gain to price changes. This analogy is based on the fact that one way to lower DMT is to increase mature weight. Since increasing mature weight is opposite to the indicated reaction to lower margins, the value of DMT falls as margins fall.

In order to compare the values of EFG and DMT, some standard measure is needed. If the unit is chosen as a standard deviation, then DMT is valued at as much as 26 times the value of a standard deviation of EFG. The dual impact of mature size on gain and DMT suggests that great attention should be given size variables by all phases of the industry; up to 26 times that given to feed efficiency, when fed to a constant maturity at slaughter.

Animal breeding has a method of determining the relative importance in selection indexes. In the context of an index, these values for EFG and DMT are the relative economic weights, a , used in equation (21). One significant difference between these values, and those previously obtained is that these values are the actual genotypic economic values. In studies such as Gibson's (9) the values are actually associated with phenotype. This means that translation into equation (21) requires the assumption that genotype and environment are uncorrelated. If this

assumption is made then phenotypic values are a good approximation of genotypic values. However, if this assumption is not true, then a method of estimating economic genotypic value must be used. The use of large cattle, which therefore consume more, is an example in which the assumption of zero genetic and environmental correlation breaks down. For these cases, the estimation method presented here is generally superior.

It is hoped that results of this and similar studies will allow producers greater freedom in animal evaluation. In this manner, traits can be used rather than animal evaluation and, thereby, improve feedlot income. It is also hoped that this study presents not only a combination of two disciplines, but an alternative method of determining economic weights for animal breeding selection indexes. The combination of these two aspects will hopefully lead to continued and increased progress in beef improvement.

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APPENDIX A. DERIVED PROPERTIES OF THE
PRODUCTION FUNCTION

Table A-1. Isoquant schedules and marginal rates of substitution of days on feed (T) for TDN (E) for Hereford steers (HH) fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$
120	816.6	1.02	1203.3	1.50	1625.4	2.03				
180	768.5	.64	1132.4	.94	1529.6	1.27	1955.1	1.63	2178.3	1.81
240	736.1	.46	1084.6	.68	1465.1	.91	1873.1	1.17	2086.4	1.30
300	711.9	.36	1049.0	.52	1461.9	.73	1811.5	.90	2017.8	1.01
360	692.7	.29	1020.7	.43	1378.8	.57	1762.7	.73	1963.4	.82
420	676.9	.24	997.4	.36	1347.3	.48	1722.5	.61	1918.6	.68

^aMarginal rate of substitution of time for TDN, MRS_{TE} , actually the absolute value of MRS_{TE} .

Table A-2. Isoquant schedules and marginal rates of substitution of days on feed (T) for TDN (E) for Hereford x Angus steers (HA) fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$
120	821.9	1.03	1211.1	1.51	1636.0	2.04				
180	773.5	.64	1139.7	.95	1539.6	1.28	1968.3	1.64	2192.4	1.82
240	740.9	.46	1091.7	.68	1474.6	.92	1885.3	1.18	2100.0	1.31
300	716.5	.36	1055.8	.53	1426.1	.71	1823.3	.91	2030.9	1.01
360	697.2	.29	1027.3	.43	1387.7	.58	1774.2	.74	1976.2	.82
420	681.3	.24	1003.9	.36	1356.0	.48	1733.7	.62	1931.1	.69

^aMarginal rate of substitution of time for TDN, MRS_{TE} , actually the absolute value of MRS_{TE} .

Table A-3. Isoquant schedules and marginal rates of substitution of days on feed (T) for TDN (E) for Angus x Hereford steers (AH) at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$
120	818.6	1.02	1206.2	1.51	1629.3	2.03				
180	770.4	.64	1135.1	.94	1533.3	1.28	1960.3	1.63	2183.6	1.82
240	737.9	.46	1087.3	.68	1468.6	.92	1877.6	1.17	2091.5	1.31
300	713.6	.36	1051.5	.53	1430.4	.71	1815.9	.91	2022.7	1.01
360	694.4	.29	1023.2	.43	1382.1	.58	1767.0	.74	1968.2	.82
420	678.5	.24	999.8	.36	1350.6	.48	1726.6	.62	1923.3	.69

^aMarginal rate of substitution of time for TDN, MRS_{TE} , actually the absolute value of MRS_{TE} .

Table A-4. Isoquant schedules and marginal rates of substitution of days on feed (T) for TDN (E) for Angus steers (AA) fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$
120	960.9	1.20	1415.9	1.77	1912.5	2.39				
180	904.3	.75	1332.4	1.11	1799.8	1.50	2332.2	1.94	2563.1	2.13
240	866.1	.54	1276.2	.80	1723.9	1.08	2233.9	1.39	2455.0	1.53
300	837.7	.42	1234.2	.62	1667.2	.83	2160.4	1.08	2376.3	1.19
360	815.1	.34	1201.0	.50	1622.3	.68	2102.2	.87	2310.3	.96
420	796.5	.28	1173.6	.42	1585.3	.57	2054.2	.73	2257.6	.81

^aMarginal rate of substitution of time for TDN, MRS_{TE} , actually the absolute value of MRS_{TE} .

Table A-5. Isoquant schedules and marginal rates of substitution of days on feed (T) for TDN (E) for all steers fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$	kg. TDN	$\frac{\partial E}{\partial T}$
120	884.8	1.10	1303.7	1.63	1761.0	2.20				
180	832.6	.69	1226.9	1.02	1657.3	1.38	2118.8	1.76	2360.1	1.96
240	797.5	.50	1175.1	.73	1587.4	.99	2029.4	1.27	2260.5	1.41
300	771.3	.39	1136.5	.57	1535.2	.77	1962.7	.98	2186.2	1.09
360	750.5	.31	1105.9	.46	1493.8	.62	1909.8	.79	2127.3	.89
420	733.4	.26	1080.7	.39	1459.7	.52	1866.2	.62	2078.8	.74

^aMarginal rate of substitution of time for TDN, MRS_{TE} , actually the absolute value of MRS_{TE} .

Table A-6. Marginal productivity of TDN (E) and days on feed (T) for Hereford steers (HH) fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\frac{\partial g}{\partial E}^a$	$\frac{\partial g}{\partial T}^b$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$
120	.138	.141	.125	.188	.116	.235	0	0	0	0
180	.147	.094	.133	.125	.123	.156	.115	.187	.112	.203
240	.153	.070	.139	.095	.128	.116	.121	.142	.117	.152
300	.159	.057	.143	.074	.132	.096	.125	.113	.121	.122
360	.163	.047	.147	.063	.136	.078	.128	.093	.125	.103
420	.167	.040	.151	.054	.140	.067	.131	.080	.127	.086

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of days on feed, MP_T .

Table A-7. Marginal productivity of TDN (E) and days on feed (T) for Hereford x Angus steers (HA) fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\partial g^a / \partial E$	$\partial g^b / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$
120	.131	.135	.118	.178	.109	.222	0	0	0	0
180	.139	.089	.126	.120	.116	.148	.109	.179	.106	.193
240	.145	.067	.131	.089	.121	.112	.114	.134	.111	.145
300	.150	.054	.136	.072	.125	.089	.118	.107	.115	.116
360	.154	.045	.139	.060	.129	.075	.121	.090	.118	.097
420	.158	.039	.143	.051	.132	.063	.124	.077	.120	.083

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of days on feed, MP_T .

Table A-8. Marginal productivity of TDN (E) and days on feed (T) for Angus x Hereford steers (AH) fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\partial g^a / \partial E$	$\partial g^b / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$
120	.140	.142	.126	.191	.117	.237	0	0	0	0
180	.148	.095	.134	.126	.124	.159	.117	.190	.113	.206
240	.155	.071	.140	.095	.130	.119	.122	.142	.118	.155
300	.160	.058	.145	.077	.134	.095	.126	.115	.122	.124
360	.165	.048	.149	.064	.138	.080	.129	.096	.126	.103
420	.169	.040	.152	.055	.141	.068	.132	.082	.129	.089

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of days on feed, MP_T .

Table A-9. Marginal productivity of TDN (E) and days on feed (T) for Angus steers (AA) fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\partial g / \partial E^a$	$\partial g / \partial T^b$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$	$\partial g / \partial E$	$\partial g / \partial T$
120	.122	.146	.110	.195	.102	.243	0	0	0	0
180	.129	.097	.117	.130	.108	.162	.101	.196	.099	.210
240	.135	.073	.122	.098	.113	.122	.106	.147	.103	.158
300	.140	.059	.126	.078	.117	.097	.109	.118	.107	.127
360	.143	.049	.130	.065	.120	.082	.112	.098	.110	.105
420	.147	.041	.133	.056	.123	.070	.115	.084	.112	.091

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of days on feed, MP_T .

Table A-10. Marginal productivity of TDN (E) and days on feed (T) for all steers fed at five gain levels (other variables at line mean)

Days on Feed	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\frac{\partial g^a}{\partial E}$	$\frac{\partial g^b}{\partial T}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial T}$
120	.126	.138	.114	.186	.105	.232	0	0	0	0
180	.134	.092	.121	.123	.112	.154	.105	.185	.102	.200
240	.140	.070	.126	.092	.117	.116	.110	.139	.107	.150
300	.144	.056	.131	.074	.121	.093	.113	.111	.110	.120
360	.148	.046	.134	.062	.124	.077	.117	.092	.113	.101
420	.152	.039	.137	.054	.127	.066	.119	.074	.116	.086

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of days on feed, MP_T .

Table A-11. Isoquant schedules and marginal rates of substitution of genetic feed efficiency (EFG) for TDN (E) for Hereford steers (HH) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$
3.5984	736.1	126.28	1084.6	186.06	1465.1	251.34	1873.1	321.33	2086.4	357.92
3.2251	688.0	131.69	1013.7	194.03	1369.3	262.09	1750.6	335.08	1950.0	373.25
2.8518	637.7	138.04	939.6	203.39	1269.2	274.73	1622.6	351.23	1807.4	391.24

^aMarginal rate of substitution of feed efficiency for energy, $MRS_{\text{EFG},E}$, actually the absolute value of $MRS_{\text{EFG},E}$.

Table A-12. Isoquant schedules and marginal rates of substitution of genetic feed efficiency (EFG) for TDN (E) for Hereford x Angus steers (HA) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$
4.0007	740.9	114.32	1091.7	168.45	1474.6	227.53	1885.3	290.90	2100.0	324.03
3.4565	676.9	120.89	997.5	178.15	1347.4	240.64	1722.6	307.65	1918.7	342.67
2.9123	613.5	130.04	894.9	189.69	1208.8	256.22	1545.4	327.57	1721.4	364.88

^aMarginal rate of substitution of feed efficiency for energy, $\text{MRS}_{\text{EFG},\text{E}}$, actually the absolute value of $\text{MRS}_{\text{EFG},\text{E}}$.

Table A-13. Isoquant schedules and marginal rates of substitution of genetic feed efficiency (EFG) for TDN (E) for Angus x Hereford steers (AH) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$
3.6957	737.9	123.25	1087.3	181.62	1468.6	245.31	1877.6	313.62	2091.5	349.35
3.1010	662.1	131.80	975.7	194.23	1317.9	262.35	1684.9	335.41	1867.8	371.82
2.5063	580.9	143.08	855.5	210.71	1155.9	284.70	1477.4	363.89	1645.6	405.32

^aMarginal rate of substitution of feed efficiency for energy, $MRS_{\text{EFG},E}$, actually the absolute value of $MRS_{\text{EFG},E}$.

Table A-14. Isoquant schedules and marginal rates of substitution of genetic feed efficiency (EFG) for TDN (E) for Angus steers (AA) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$
4.2893	866.1	124.65	1276.2	183.67	1723.9	248.10	2233.9	321.50	2455.0	353.32
3.8459	809.7	129.97	1193.1	191.51	1611.6	258.68	2060.4	330.72	2295.1	368.39
3.4025	750.7	136.20	1106.2	200.70	1494.2	271.09	1910.3	346.58	2127.9	386.06

^aMarginal rate of substitution of feed efficiency for energy, $\text{MRS}_{\text{EFG},\text{E}}$, actually the absolute value of $\text{MRS}_{\text{EFG},\text{E}}$.

Table A-15. Isoquant schedules and marginal rates of substitution of genetic feed efficiency (EFG) for TDN (E) for all steers at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain ^a		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$	TDN	$\frac{\partial E}{\partial \text{EFG}}$
4.0921	797.5	120.31	1175.1	177.27	1587.4	239.47	2029.4	306.14	2260.5	341.01
3.166	680.7	132.72	1003.0	195.57	1354.8	264.16	1732.1	337.73	1929.4	376.20
2.2399	549.8	151.52	810.1	223.26	1094.3	301.59	1399.0	385.56	1558.3	429.46

^aMarginal rate of substitution of feed efficiency for energy, $MRS_{\text{EFG},E}$, actually the absolute value of $MRS_{\text{EFG},E}$.

Table A-16. Marginal productivity of TDN (E) and genetic feed efficiency (EFG) for Hereford steers (HH) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\frac{\partial g^a}{\partial E}$	$\frac{\partial g^b}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$
3.5984	.153	-19.321	.139	-25.862	.128	-32.172	.121	-38.881	.117	-41.877
3.2251	.164	-21.605	.148	-28.804	.137	-36.004	.129	-43.206	.125	-46.808
2.8518	.177	-24.433	.160	-32.575	.158	-43.514	.139	-48.861	.135	-52.935

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of genetic feed efficiency, MP_{EFG} .

Table A-17. Marginal productivity of TDN (E) and genetic feed efficiency (EFG) for Hereford x Angus steers (HA) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\frac{\partial g}{\partial E}^a$	$\frac{\partial g}{\partial EFG}^b$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$
4.0007	.145	-16.576	.131	-22.067	.121	-27.531	.114	-33.163	.111	-35.967
3.4565	.164	-19.773	.148	-26.367	.137	-32.959	.129	-39.550	.125	-42.844
2.9123	.181	-23.598	.165	-31.229	.152	-39.036	.143	-46.843	.139	-50.747

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of genetic feed efficiency, MP_{EFG} .

Table A-18. Marginal productivity of TDN (E) and genetic feed efficiency (EFG) for Angus x Hereford steers (AH) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\frac{\partial g}{\partial E}^a$	$\frac{\partial g}{\partial EFG}^b$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$
3.6957	.155	-19.104	.140	-25.427	.130	-31.890	.122	-38.262	.118	-41.223
3.1010	.168	-22.158	.152	-29.548	.141	-36.934	.132	-44.320	.129	-47.784
2.5063	.192	-27.430	.173	-36.558	.161	-45.707	.151	-54.838	.147	-59.406

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of genetic feed efficiency, MP_{EFG} .

Table A-19. Marginal productivity of TDN (E) and genetic feed efficiency (EFG) for Angus steers (AA) at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\frac{\partial g^a}{\partial E}$	$\frac{\partial g^b}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$
4.2893	.135	-16.828	.122	-22.408	.113	-28.035	.106	-34.079	.103	-36.392
3.8459	.137	-17.868	.124	-23.824	.115	-29.780	.108	-35.736	.105	-38.715
3.4025	.148	-20.196	.134	-26.929	.124	-33.660	.117	-40.391	.113	-43.759

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of genetic feed efficiency, MP_{EFG} .

Table A-20. Marginal productivity of TDN (E) and genetic feed efficiency (EFG) for all steers at five levels of gain (T=240, DMT=line mean)

EFG Level	150 kg. gain		200 kg. gain		250 kg. gain		300 kg. gain		325 kg. gain	
	$\frac{\partial g^a}{\partial E}$	$\frac{\partial g^b}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$	$\frac{\partial g}{\partial E}$	$\frac{\partial g}{\partial EFG}$
4.0921	.140	-16.843	.126	-22.336	.117	-28.018	.110	-33.675	.107	-36.488
3.1660	.164	-21.705	.148	-28.941	.137	-36.174	.129	-43.410	.125	-47.029
2.2399	.202	-30.680	.183	-40.906	.170	-51.135	.159	-61.360	.155	-66.472

^aMarginal productivity of TDN, MP_E .

^bMarginal productivity of genetic feed efficiency, MP_{EFG} .

APPENDIX B. GLOSSARY OF TERMS

Animal Breeding Terminology and
Abbreviations for Economists

<u>Term</u>	<u>Symbol/Abbreviation</u>	<u>Definition</u>
Aggregate breeding value	H	The overall breeding value; sum of breeding values for each trait multiplied by economic values due to multiple trait selection.
Average effect of gene substitution	α	Change in population mean value due to substitution of one gene for another.
Breeding value	A	The value of an individual's progeny above the mean of the population.
Crossbreeding/Inbreeding		The mating of individuals more (inbreeding) or less (crossbreeding) closely related than the average of the population.
Environment		All nongenetic aspects of an individual's production.
Genes	$A_1; A_2$	The basic unit of inheritance.
Genotype	$A_1A_1; A_1A_2; A_2A_2$	The combination of genes in an individual; one gene of each pair comes from each parent.
Genotypic values	a;d	The measurement (value) associated with a genotype on a trait.
Heritability	h^2	The ratio of additive genetic variance to phenotypic variance $\frac{V(A)}{V(P)}$.

<u>Term</u>	<u>Symbol/Abbreviation</u>	<u>Definition</u>
Heterozygote	$A_1 A_2$	The individual expressing different genes.
Homozygote	$A_1 A_1 ; A_2 A_2$	The individual with like genes at a locus.
Intensity of selection	i	The number of standard deviations of phenotypic value selected above the population mean.
Phenotype		The observed expression of a genotype.
Phenotypic value		The measurement (value) associated with a phenotype in traits.
Population		Reproductive individuals sharing a common gene pool.
Response	R	The change in population mean values due to selection.
Selection		The process of allowing certain individuals to reproduce at higher rates than others.
Selection differential	S	The difference in superiority of selected parents and the average of the population from which they came.
Selection index	I	A weighting system for multitrait selection.

Economic Terminology and Abbreviations
for Animal Scientists

<u>Term</u>	<u>Symbol/Abbreviation</u>	<u>Definition</u>
Average productivity	AP_i	Total units of output divided by total units of i th input Y/X_i .
Average variable cost	AVC	Total variable cost of production divided by units of output TVC/Y .
Elasticity of production	e_i	Percentage change in output due to a one percent change in the i th input; all other inputs fixed-- $\frac{\partial Y}{\partial X_i} \cdot \frac{X_i}{Y}$.
Elasticity of substitution	s_{ij}	Percentage change in the use of the i th input due to a one percent change in the j th input use.
Expansion path		The set of optimal production points as the production increases over time.
Factor marginal cost	FMC	The cost of the final unit of an input used.
Isocline		The set of all points of production such that the marginal rate of substitution between inputs is the same.
Isoquant		The set of all input combinations which will produce a given output.

<u>Term</u>	<u>Symbol/Abbreviation</u>	<u>Definition</u>
Marginal cost	MC	The added cost of the last unit of output produced.
Marginal productivity	MP_i	The contribution to output of the last unit of the i th input used-- $\partial Y / \partial X_i$.
Marginal rate of substitution	MRS_{ij}	The rate at which the last unit of the j th input will substitute for the i th input-- $\partial X_i / \partial X_j$.
Marginal revenue	MR	The contribution to total revenue made by the last unit of output.
Marginal value product also Value of marginal product	MVP_i	The value of the output produced by the last unit of the i th input (MR) (MP_i).
Profit	π	Total revenue less total cost TR-TC.
Ridgeline		The boundary of a set of isoquants where the marginal rate of substitution equal zero or infinity.
Scale effect also Returns to scale	r	The percentage change in output if all inputs were simultaneously increased by one percent.
Total Cost	TC	All costs associated with production.
Total cost-fixed	FC	Costs which cannot be escaped with zero production.

<u>Term</u>	<u>Symbol/Abbreviation</u>	<u>Definition</u>
Total cost-variable	VC	Costs which vary with production.
Total revenue	TR	All revenue associated with production.